STRUCTURAL OPTIMIZATION OF MULTIMODAL ROUTES FOR CARGO DELIVERY

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Abstract:
This article is devoted to the coordination of single stages of the multimodal delivery process, taking into account the fact that the process is discrete in its content. The fact, which has the content of a time window for performing the operation is used for discrete processes. Due to the fact that multimodal transportation of goods is carried out on a large network, time is one of the most important criteria for their perfection. Two timing criteria are applied in the article, which take into account the fact that the multimodal process must be synchronized and that the transportation of a large group of goods can be carried out in separate parts. An estimation criterion was also applied, which takes into account constant, variable, contingent costs, which are carried out depending on the structure of the process. The goal of the study is to create such multimodal cargo delivery routes that are characterized by the highest level of selection criteria. In contrast to known studies, the dependence of the optimization criteria of the multimodal process on the total volume of cargo delivery was shown. The method of analyzing the transport scheme of multimodal transportation and the corresponding algorithm and computer program were developed. The methodology involves a complete review of all possible route options using three types of continent transport, namely road, rail, and river. The method of structural optimization is applied to the example of a transcontinental transport corridor.

Keywords: multimodal transportation, discrete processes, structural optimization, process cycle

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1. Introduction

Multimodal (mixed) cargo transportation is characterized by increasing volumes in Europe and Asia due to its main advantages, which have become more relevant taking into account the growth of cargo turnover and the increase in the average distance of 1 ton of cargo transportation. These advantages include a decrease in the share of automobile transportation in routes, which is associated with the use of more ecological types of transport (railway, river). It is difficult to ensure working and rest conditions for drivers of trucks and road trains on long-distance road routes. Multimodal transportation partially solves this problem. On the other hand, road transportation as part of multimodal transportation is often irreplaceable due to the much greater availability of the road network and faster delivery speed than other modes of transportation. Rail transportation is characterized by low environmental pollution, lower delivery risks and significantly lower costs per unit of cargo, but it requires a lot of time for preparatory and final operations, is less flexible in terms of work schedule and, accordingly, less accessible on the transport network. Water transport, in particular river transport, provides large volumes of solid cargo, is the least energy-consuming, but is characterized by low delivery speed, and is highly dependent on natural and climatic conditions. All three types of transport have similar mutual advantages and disadvantages. Therefore, they are logically combined in a single delivery process between given transport hubs and terminals so that the overall positive effect of the combination is maximal. In some cases, the negative features of various types of transport can be leveled thanks to certain conditions of combination and coordination. This article is devoted to identifying the conditions for the effective use of multimodal transportation, because such conditions remain insufficiently researched.

The development of transport systems in Europe and Asia makes it possible to choose different combinations of routes thanks to the development of relevant networks, the growth of the number of vehicles and the development of infrastructure. On the other hand, the conditions and limitations of the delivery of goods are unified. Against the background of a large selection, the share of international delivery standards is increasing. The rational choice of a package of decisions regarding the multimodal route of delivery of unified cargo should be based on the technical analysis of individual components, justification of their choice and coordination. The final decisions should be justified not only with respect to the qualitative selection of multi-route elements, but also have numerical criteria. The main expected result is aimed at evaluating different options for multimodal transnational routes, taking into account the volume of cargo transportation, the duration of delivery and the economic component in the form of specific costs for delivery. The purpose of the study is to create such multimodal cargo delivery routes that are characterized by the highest level of selection criteria. The goal is achieved by selecting such parameters of partial cargo flows that lead to their best coordination. Research tasks that led to the achievement of the goal:

a) on the basis of the theoretical model of discrete cargo flows, determine the signs and conditions of optimality of a multimodal route;

b) develop a method of analysis and selection of the most rational among competing multimodal routes and verify its effectiveness on a practical problem.

2. Literature review

It is claimed by (Grznar et al., 2021) that multimodal (mixed) cargo transportation is aimed at eliminating regulatory, technical, organizational, financial and other barriers in the process of cargo moving. That is, the main necessary conditions in the organization of multimodal (mixed) transportation are the presence of three elements: a regulatory and legal field for carrying out such transportation; technical means; corresponding infrastructure (Cieśla and Opasiak, 2021). However, the presence of only prerequisites does not eliminate the contradictions that arise when coordinating individual operations of a multimodal process. One of the reasons for the insufficient development of the share of multimodal transportation is their high requirements for the coordination of individual transport services. In order to obtain an obvious advantage from the use of multimodal transportation, it is necessary to consider and substantiate their structure and relevant...
organizational parameters. The transport network and geographical conditions are the most important factors of multimodal transportation, according to (Wang, Y. et al., 2022). The cargo delivery network is significantly distributed in space. This means that one of the main factors affecting its efficiency is time, as well as fluctuations in shipment volumes, delays and failures of transport systems within the network. In the paper of (Akyüz et al., 2023) the problem of replanning intermodal routes is formulated as the arrangement of flows of a multi-commodity transport network. According to this approach, the authors considered two different network topologies, one of which is based on the temporal-spatial aspect, and the other embeds the temporal aspect in a highly scalable alternative structure for large transport networks. The problem of process reorganization is considered when there is an unplanned need to find an alternative transportation if there are failures or delays in the network. Criterion of optimal solutions (Akyüz et al., 2023, Ambrosino and Sciomachen, 2014, Archetti et al., 2022, Hochbaum and Levin, 2006) there are general transportation costs. However, the costs of cargo movement also depend on the volume of the performed transportation, which was not taken into account in the previous works. By the authors (Beresford et al., 2021) the impact of changes in the volume of transportation on costs is not taken into account at all. Adherence to time tolerances for transportation is also not considered. This means that under certain circumstances the recommended solutions may not be optimal. Instead, at work of (Wang, Y. et al., 2022) it was noted that a multimodal transportation network has more stringent requirements for compliance with time limits and recommendations than a single mode of transport. The shortest duration of transportation is a general goal when designing or optimizing the path of a multimodal transportation network. However, the calculation of the delivery time of integrated systems often does not coincide with real situations. According to these authors, the main error is the occurrence of flow pulses, time irregularities of the process, therefore a dynamic routing model was developed, which is based on the use of a large database. However, with such cumbersome calculations, the error can increase even more. The authors did not reveal the reasons for this phenomenon.

The duration of delivery on individual sections of a multimodal route often does not correspond to the "just in time" principle. If the cargo arrives at the transshipment point early, then it is necessary to wait until the contact transport arrives at the point. And this requires additional storage costs. If the shipment arrives late, fines will be imposed, or the next shipment period will be missed (especially by sea). In other words, too early or late delivery of the cargo will bring economic losses, which can be measured with the help of discount factors. The authors (Karimi and Bashiri, 2018) consider one of the reasons for the inconsistency in fixing hierarchical relationships between the main carriers and the need for decision-making by carriers in maritime freight transportation networks. Carriers make pricing and routing decisions in every part of the intermodal freight system and have consistent relationships. However, technological and organizational causes of non-synchronism were not considered in the article. The uncertainty of the duration of transportation operations between nodes is a significant obstacle to perform route optimization of a multimodal transportation network to guarantee reliable delivery times (Lunardi et al., 2020, Oliskevych, 2018, Oliskevych et al., 2022, Peng et al., 2022). The stochasticity of the processes (Naumov et al., 2020) also leads to unsuccessful attempts to build a single-valued optimal schedule of multimodal cargo delivery (Castelli et al., 2004, Hochbaum and Levin, 2006). Most of the optimization problems of multimodal transport routes are stable with respect to the design conditions. However, the requirements of customers and transport companies were often diverse and even contradictory (Kézai et al., 2020, Macioszek et al., 2023). Therefore, previous studies can only provide multimodal carriers with a guideline for decision-making when making a transportation plan under conditions of uncertainty. However, it does not indicate how the influence of random conditions on the reliability of decisions can be reduced. The issues of synchronization of various operations in time are also not considered by these authors. Methods of synchronizing of individual flows in an integrated network were known in the practice of
intermodal transportation. In particular, one of the most successful is SYNCHRO-NET. It is a project consisting of several modules to optimize the integrated management of freight transportation and logistics at strategic, tactical, operational and real-time levels (Ambrosino and Sciomachen, 2014, Giusti et al., 2019). SYNCHRO-NET is aimed at optimizing the simultaneous interaction of several interested parties, that is, customers, carriers, terminals, intermediaries. The final selection is made simultaneously under two conditions. In particular, the synchronization of schedules between different parts of the route should be ensured and the use of different types of transport should be coordinated. The project uses three different criteria at the same time (traveled distance, time, and CO2 emissions). Such problems presented in SYNCHRO-NET are NP-complete in general and can be solved only using modified heuristic algorithms. In some cases, the optimal solution is not guaranteed. More accurate and guaranteed optimal solutions are obtained in the case of decomposition of the general problem into the problem of optimization of chains of separate deliveries of cargo groups (Taran et al., 2023). Many studies have been conducted on the cost of multimodal transport (Archetti et al., 2022, Castelli et al., 2004, Zhang, X. et al., 2021, Zhang, H. and Li et al., 2021). But most of them only take into account the transport fee and the cost of transit. Very few researchers divide the multimodal transport process into several operations (Akyüz et al., 2023, Baykasoğlu et al., 2019, Beresford et al., 2021). Costs are roughly quantified without reflecting the exact proportion of remuneration for each link of multimodal transport. The costs associated with the temporal inconsistency of the process are also not taken into account.

It can be seen that the geographical location of nodes plays a decisive role in forecasting the volume of cargo, selecting nodes and optimizing transport routes in a multimodal transport network. For example, knowing the geographical location, a company can easily find the most economical transport route or a storage station for goods. The influence factors and characteristics of multimodal transport behavior were investigated at work of (Wang, Y. et al., 2022) based on logit models and large volumes of various data. This enabled the authors to achieve the optimal effect of freight transportation by properly reducing the share of road transport and increasing the share of other types of transport (Fischer, 2021). However, researchers miss the indisputable advantages of automobile delivery of goods at certain stages of multimodal transportation, in particular, efficiency. Again, the effect of improving multimodal transportation depends on the structure of routes and the volume of transportation, which the authors did not fully consider.

Logistics network design is an abstract optimization problem. Despite this, such a problem is solved according to the criterion of minimum costs. Junctions are most often represented in the form of an optimal infrastructure configuration (Archetti et al., 2022, Grznar et al., 2021, Fan et al., 2022, Lada et al., 2016, Udomwannakhet et al., 2018). Its solutions do not have a direct practical application, since it is not possible to change the established nodes of the transport network. Most likely, the solutions obtained from the application of cost-time-distance models offer a systematic view of the relationships between modes of transport, nodes, methods and volumes, types and forms of cargo. The revised multimodal transportation cost model is based on a relatively simple framework, but demonstrates that other existing models of mode choice, multimodal transportation, and inventory placement oversimplify the transportation process. The authors (Archetti et al., 2022) indicate the ambiguity of decisions regarding the choice of the configuration of multimodal routes and confirm the large non-linearity of the corresponding dependencies.

If land transport suffers from delivery delays, accidents, environmental pollution, cargo loss, then the main problem of water transport in the supply chain is the lack of space for storing containers. In fact, seaports are now suffering from a lack of space at marine terminals and increasing congestion on access roads to inland connections (Ambrosino and Sciomachen, 2014). The current problem fits into the class of problems of determining the location of several nodes and terminals. However, such a problem is not solved together with others included in the chain of multimodal transportation. The authors (Castelli et al., 2004) also confirm that the location
of intermediate transport nodes are important elements of multimodal freight distribution networks and this largely affects their efficiency, overall network management, flow routing decisions and the choice of transportation mode. Calculating the global minimum (maximum) of a function of many variables, which are the coordinates of network nodes, is usually a difficult problem. The objective function is nonlinear, and the problem belongs to the NP-hard complexity class (Sun, Y. 2022). There is a large list of classic algorithms for optimization problems that are heuristic or metaheuristic. The article used the annealing algorithm. This is the most successful solution in this case, however, the obtained solutions apply only to stationary cargo flows of the transport network. When changing streams, the solutions obtained cannot be recommended. The application of mathematical programming methods in order to increase the accuracy of the guaranteed solution of the multimodal route optimization problem is debatable. A mixed integer linear programming model is presented in article (Ambrosino and Sciomachen, 2021), which aims to minimize location and delivery costs while accounting for external cost components. However, the authors did not take into account the non-linear nature of the problem. It is necessary to take into consideration the capacity of cargo flows on the network, which may change. The obtained solutions are static, as for the previous cases when heuristic methods are applied. The imperfection of the methods used in solving problems of multimodal transportation is a typical drawback. Thus, a multi-commodity multi-modal supply chain network is being developed for intelligent manufacturing at work (Karimi and Bashiri, 2018). This study aims to find the optimal location of transport hubs and other network elements to ensure efficiency. However, to minimize the total cost of logistics, a mixed integer linear programming model is proposed as a compromise between mathematical programming methods and heuristic methods. At the same time, the non-linear dependence of variable costs on the capacity of the cargo flow, the random nature of the processes, as well as the lack of alternative modal routes are not taken into account. The problem of optimizing the route of vehicles using an adaptive genetic algorithm, which is based on Monte Carlo sampling, is studied in the article (Zhang, H. et al., 2021). However, its successful application on multimodal networks requires large volumes of input data. It is also necessary to take into account the variability of the initial optimization conditions, which makes the obtained solutions unstable. The main problems of the development of multimodal transportation, formulated by the authors (Lee and Choo, 2016) are proposed to be solved by heuristic or metaheuristic methods in connection with problem nonlinearity. At the same time, the authors cite the development of new intelligent technologies for coordination of different types of transport as a factor that pushes the need for further research in this area. The problems continue to worsen along with the development of modern methods. These include, in particular, an increase in traffic jams at network nodes, a rapid increase in costs with an increase in freight flows, and difficulties in implementing synchronomodal transportation. Article of (SteadieSeifi et al., 2014) presents a structured review of the literature on the chronology and correlation of multimodal transportation management problems and methods since 2005. The traditional strategic, tactical, and operational levels of planning are described in detail. It is characteristic that the greatest attention in the literature is paid to networks with a "hub-and-spoke" topology, where rail and road transport prevail. On the other hand, the network of corridors, where water transport is more common, has not been developed sufficiently. As for transport resources, there is also a large gap in the research of forward and reverse direct freight flows. At (Wang, Y. et al., 2022) it is shown that due to the integration of individual optimal solutions for repositioning transport nodes in the network, the global design solution of multimodal networks is significantly more efficient and saves the time of researchers and managers. In addition, due to the limited number and capacity of connecting routes, as well as additional rules and restrictions (for example, the rules of working hours for drivers), it is necessary to perform planning based on universal estimates of several resources. Taking into account the dynamism and stochasticity of the data (Monk and
Fischer, 2023) also remains a major research problem. It is obvious that interaction and competition between carriers affect the execution of plans. Their cooperation, for example, ensures timely delivery. In addition, the integration of different levels of planning can provide greater reliability, flexibility and, more importantly, sustainability, creating more efficient solutions for the industry. For adequate planning of multimodal transportation, it is important to adapt to emerging disturbances. A new mathematical model of linear distribution of containers with time constraints is proposed in the article (Van Riessen et al., 2015). This model is used to determine the impact of three main types of transit disruptions on network performance: early departures, late departures and domestic flight cancellations. It is applied on the example of the European Gateway Services network. This network is developed and operated by EUROPE CONTAINER TERMINALS (ECT). For the expedient economic use of this network, centralized planning by the sea terminal of container transportation is required. However, significant fluctuations in cargo flows lead to underloading of containers and vehicles that deliver them to end consumers. The impact of various obstacles is measured in two ways. The first method is based on the difference between planned and actually incurred expenses. However, at the same time, the opportunity to record the location of the malfunction is lost. The second method is based on the fact that the network operator can select focus areas to prevent high-impact disturbances and improve planning updates for high-relevance failures. However, both methods do not reflect a systemic analysis of the network's functioning, and accordingly, the decisions are not of a systemic nature. Works are devoted to the issue of calculating the cost of multimodal transportation (Beresford et al., 2021, Castelli et al., 2004, Van Riessen et al., 2015). The authors mainly used statistical analysis to determine all possible route options, dividing them into component parts, calculating the cost of each of them, and obtaining a general assessment of the profitability/unprofitability of transportation. At the same time, all possible variants of tours that exist in practice were considered. However, there are unknown works where the costs incurred for transportation were tied to the scale of transportation. The authors analyzed the factors affecting transport costs, in particular their constant and variable components. There are different formulas for calculating these parameters. The formulas are based on the experience of cost estimation by logisticians and dispatchers of large transport companies. As a result of each of the well-known studies, the authors propose a new methodology for calculating the cost of transportation based on the statistical analysis of flights. Therefore, it is worth expressing doubts about the complete objectivity of the calculations. We can, however, hope that further research, accumulation and analysis of statistical data on the costs of various options for multimodal transportation of specific transport companies will make it possible to identify stable patterns, taking into account the accumulated experience in the statistics of a specific transport company (Van Riessen et al., 2015). The importance of using time windows and large databases for objective assessment and planning of high-quality multimodal delivery has been thoroughly proven in the work of (Wang, S. and Fu, 2022). Multiple windows are used to increase customer flexibility and get service on time. The flexibility of road service and, as a result, the optimization of truck operations are investigated by combining the scheduling of truck departures under traffic constraints and the optimization of speed with the route. In order to increase the feasibility and optimality of the problem solution, the routing problem is formulated in a fuzzy environment, where bandwidth and tariffs for resources and services are trapezoidal fuzzy parameters. The combined functions of route design, vehicle distribution, time window selection, and rolling stock optimization are assigned to the model. However, the structure of the transport process as a whole is not reflected in the work, which does not make it possible to determine its properties on the basis of cause-and-effect relationships.

An effective algorithm for solving the problem of multimodal traffic distribution on the example of a city transport multi-network is proposed in (Grznar et al., 2021, Fan et al., 2022). An analogy with a large-scale transport network can be drawn based on a discrete approach to traffic modeling. The authors showed that traffic delays are a direct consequence of the asynchrony of fixed moments in the...
journey of network users. A heuristic procedure based on Lagrange for planning the transport schedule of multimodal transport networks is presented in the article (Castelli et al., 2004). This is one of the few attempts to show the properties of the network under the dynamic effect of disturbances and changes in the requirements for the parameters of goods delivery. At the same time, the configuration of routes and the quality of delivery remain constant. And Lagrangian multipliers were used to build sustainable schedules of the delivery system. When Lagrangian relaxation is performed using multipliers and after some regrouping of variables, a problem can be obtained, which consists of a set of subproblems, very similar to the problem of dynamic replenishment of the common stock when the volume of requirements changes. However, it can be agreed that no analytical mathematical model can fully capture the economic and social consequences of meeting delivery schedules for complex transport networks, which are multimodal. Therefore, means of operational intervention should be provided in the management of delivery processes. They can be developed on the basis of structural analysis of processes.

The general problem of substantiating the power of a multimodal network is solved in the paper of (Wang, Y. et al., 2022). The authors developed a three-level problem of designing a multimodal network in order to maximize its throughput. The models are solved using an efficient optimization algorithm based on the surrogate Kriging model in urban transport networks. The obtained results demonstrate the high productivity of the method. However, this method cannot be applied to multimodal long-distance/transnational networks, since the duration of one delivery cycle on an urban network is not comparable to the duration of transportation, as it is on large-scale networks. Therefore, this approach needs to be modified for appropriate use.

3. Research method

Cargo flows are considered discrete in the modern methodology of transport systems, and cargo units as individual products, transport packages, containers are mostly moved in groups. This means that qualitative changes in cargo flows occur at fixed moments of time, and the flow itself has the properties of accelerating / decelerating, splitting or merging. As a matter of fact, the first two properties are implemented in direct-flow transport and technological schemes thanks to the bulk movement of goods. If a discrete material flow has an average intensity \( \mu \) of movement between given route points \( q_1 \) and \( q_2 \), and movement is carried out by a group of \( k_1 \) cargo units, then, based on the principle of flow continuity, this intensity will be preserved even when after point \( q_2 \) the flow changes its properties, and will further move with group size \( k_2 \). However, the speed of flow will change inversely proportional to the changes in the size of its bulk in this case. We will call all such changes Elementary Logistics Operations (ELO). They can be followed if the flow is divided into separate divisions, the duration of which is called a tact (Lunardi et al., 2020, Oliskevych, 2018, Oliskevych et al., 2022, Grznar et al., 2021). A cycle is a period of time between successive qualitative changes of the same material flow (cargo flow, vehicles flow). For example, one calls a tact the period between the moments of change in the size of the group \( k_i \) and \( k_j \) between the change of vehicle for the further movement of a stable group of goods \( k \), between the release of another vehicle for the movement of a given amount of transportation. It was claimed previously that the tact is characteristic only for periodic processes (Oliskevych, 2018). But soon the tact is a time window in its meaning simultaneously, within which the ELO should be performed. An ELO can be performed once during the cycle. If at the same time \( k_i < K_\Sigma \), where \( K_\Sigma \) is the total group size of goods, which have to be transported according to the given multimodal scheme, then the number of repetition cycles of ELO will be more than one. We can write

\[
\tau = \frac{k}{\mu}, \text{ hours.} \tag{1}
\]

If the tact value of two neighboring ELOs \( e_i \), \( e_j \), which refer to the same cargo flow, are different in value, \( \tau_i \neq \tau_j \), it means that these ELOs differ only in the size of the cargo group \( k_i \neq k_j \), since \( \mu = \text{const} \). Formal restrictions are imposed on the numerical value of the tact of any ELO:
\[ \tau_{\text{max}} \leq f \cdot t_i, \]  
\[ \tau_{\text{min}} = \max \left( \frac{t_i}{f_i} \right), \]  
\[ t_i \text{ – duration of the } i \text{th ELO, } f_i \text{ is the front of vehicles that simultaneously transport cargo during the execution of the } i \text{th ELO. The value of the ELO front was chosen in this way due to the fact that for the } i \text{th ELO its duration may exceed the cycle time (tact) determined by (1). In this case, several vehicles are used to transport goods, and they work according to the cyclogram (Fig. 1). It can be seen from the cyclogram that } t_i > \tau_i. \text{ At the same time, the tact is selected from (1). In order for condition (1) to be fulfilled, it is necessary to additionally ensure the ratio: } \]

\[ f = \left[ \frac{t_i}{\tau_i} \right], \]  
\[ \text{where these square brackets mean rounding to a larger whole.} \]

Thus, if } t_i = 3.8 \text{ hours, and the cycle time } \tau = 2 \text{ hours, for a sample then 2 vehicles are needed to perform ELO within the cycle time according to Fig. 1. This can be seen from the cyclogram. There will be no more than two segments in any of its cross sections. The maximum tact value } \tau_{\text{max}} \text{ follows from the condition that } \tau_{\text{max}} \leq f \cdot t_i, \text{ otherwise there is no point in setting time windows for the } i \text{-th ELO. Thus, the upper limit of the tact } \tau_{\text{max}} \text{ is set from the conditions of rational use of the time fund. Let’s consider the process of multimodal transportation as a sequence of ELOs, which are elementary discrete material flows. The process can be represented as a chain of successive arcs connecting the vertices } e_1, \ldots, e_i, e_j, \ldots, e_N, \text{ each of them is the moment of overloading the cargo to another vehicle at the corresponding node. Vertex } e_1 \text{ is the moment of cargo departure from the first point } q_1, \text{ vertex } e_N \text{ is cargo unloading at destination } q_N. \text{ Each arc } (e_i, e_j) \text{ has quantitative weights } t_{ij}, k_{ij}, f_{\text{max}}, \mu_{ij}, \text{ which mean, respectively: the time of transportation (including the duration of preparatory and final, loading and unloading actions), the maximum size of the cargo group units (packages, containers, tons) that can be transported in one trip, the maximum number of vehicles that can be used on this section of the route, the average estimated intensity of cargo transportation, calculated from (1). Separately, the time parameters of the arc may be subject to restrictions regarding the allowed time of transportation, which may be due to a fixed schedule of traffic, for example, a railway train. In this case, the parameter is specified } t_{ij}^{\text{max}} = \text{const.} \]  

The number of alternative variants of the multimodal transport and technological scheme depends on the possibility of using different types of transport, the configuration of the multimodal transport network (the presence and location of transport hubs and terminals and the communication routes between them), the capacity of individual sections of the network. Taking into account the largest transport corridors of Eurasia, trade cargo flows, it can be argued that the average number of intermediate nodes where overloading occurs and through which the desired route can be built does not exceed five to six.
The graph representing the network of alternative routes $G(Q, U)$ is an oriented graph that may contain cycles where $Q=\{q_1, q_2, \ldots, q_i, \ldots, q_N\}$, $U$ set of arcs between given vertices $Q$, any $u_{i,j}$ of them corresponds to the available connection path between $q_i$ and $q_j$, taking into account the available modes of transport. If there are several paths of connections by different modes of transport between $q_i$ and $q_j$, then, in accordance, $U$ contains multiple arcs $u_{i,j}^1$, $u_{i,j}^2, \ldots, u_{i,j}^\theta$, where $\theta$ is the total number of available modes of transport. The number of arcs $U$ depends on the real geographical data, which represent the routes of communication between transport nodes available in practice. The direction of each arc should be chosen for practical reasons. So, if there are two points $q_i, q_j$, which are connected by paths $u_{i,j}$, $u_{j,i}$, then theoretically a multimodal route can run along one or the other path. But it is practically possible to distinguish a number of technological features, according to which one of the two alternative directions of the route section is a priori unavailable: lack of transport for sending goods at point $i$ or $j$, an obvious advantage in terms of the length of the general route passing along $u_{i,j}$ than along $u_{j,i}$; lack of contact charts in one of the two points. An example of choosing alternative routes can be seen in fig. 2.

Fig. 2 shows that there are two alternative routes: 1-1-i-N and 1-j-i-N. None of them has an advantage over the other in terms of length. However, it is necessary to check the following signs: a) whether the contact transport is in the $i$-N direction in the case of choosing the 1-j-i-N route, or in the $j$-N direction in the case of choosing the 1-i-j-N route; b) does the direction of movement $j-i$ or $i-j$ comply with legal, transport regulations (does cabotage take place, is there an advantage of road/transport conditions)? In every case, when there is any advantage in choosing one of the opposite directions, it must be used, otherwise the task of finding routes cannot be solved due to the presence of cycles (Lunardi et al., 2020). Ordering chains in a directed graph is another, more complex task that was not considered in this study. The initial unordered model of the multimodal transportation process is a graph $H(E, Y)$, which displays the graph $G$, where $E=\{e_1, e_2, \ldots, e_j, \ldots, e_N\}$ is a set of vertices that represent the starting points of the corresponding ELOs, $Y$ is the set of arcs that correspond to the practical information about the movement parameters between given vertices of the graph $G$, while $e_1$ is the initial vertex of the graph $H$, the one to which no arc enters, $e_N$ is the final vertex, from which no arc from $Y$ departs. If there are multiple arcs in the graph $G$ that connect the vertices, for example, $q_i, q_j$, with the help of $n$ arcs, then $n$ such vertices as $e_{i,1}, \ldots, e_{i,n}, e_{j,1}, \ldots, e_{j,n}$ are entered in the graph $H$, between which $n$ corresponding arcs are drawn. All other vertices of the graph $H$ have at least one input and at least one output arc. It is necessary to find all possible paths from the vertex $e_1$ to the vertex $e_N$, such that only one path passes through any vertex. A subgraph $H_i$ of the graph $H$ is a simple chain, and represents one multimodal transport process. In the chains of all routes to each vertex, except $e_1, e_N$ has one entry and one exit arc. It will be the finite set of paths that reflect alternative processes, given the restrictions imposed on the graph $H$.

![Fig. 2. Route selection scheme](image-url)
The recursive search procedures were applied since the search for routes on a graph $H$ with the number of vertices $N$ is a combinatorial problem. The number of possible alternative routes from $e_1$ to $e_N$, which can include vertices $e_2$,...,$e_{N-1}$, that is, a maximum of $N-2$ vertices of the graph $H$, is determined by the ratio
\[
A_n^i = \frac{(N-2)!}{(N-2-i)!}
\]
where $i$ is the number of vertices of the graph $H$, except for $e_1$, $e_N$, which are included in the alternative route. Thus, for a sample, with $N=10$ and two fixed vertices included in each route, the number of alternative routes could be 69,280. However, due to the constraints of existing connections between pairs of vertices of graph $H$, the real number of route options is reduced to $3y^2$ times, where $y$ is the number of arcs of the graph $H$. So, if there are 15 arcs between the vertices of the graph, then the number of alternative routes is 34. This is not much for a backward time search, and the computational complexity of the algorithm for their search with a complete search of options is $O(N^{3y^2})$. It is also necessary to perform a deep search for paths in the graph $H$, which is quite small. It is advisable to use a full selection of options using recursion, therefore when choosing the optimal multimodal route.

The initial data for the application of the technique are:

a) matrix of ELO durations ($t_{ij}$), such that $t_{ij}=+\infty$ means no arc between corresponding vertices of the graph $H$;

b) matrix of type and type of transport ($\theta_{ij}$); different types of transport also include vehicles of the same type (automotive, railway, water), which have a load capacity of different ranks, for example, large-, medium- and low-tonnage trucks, river barges, ferries, etc.;

c) matrix of the maximum actual carrying capacity of vehicles ($k_{ij}$);

d) matrix of available restrictions on the number of vehicles ($f_{ij}$);

e) matrix of the available time constraints of the work schedule of vehicles ($\tau_{ij}$).

If there are no actual constraints on the arcs of the graph $H$, then the corresponding elements in the matrices are in place c), d), e) are $k_{ij}=+\infty$, $f_{ij}=+\infty$, $\tau_{ij}=+\infty$.

If there are no actual constraints on the arcs of the graph $G$, then the corresponding elements in the matrices are in place:

- $C_{c,r}$ – constant costs for transportation on one section of the route, which do not depend on the volume, distance, or time of transportation, but only on the type of vehicle;
- $C_{l,r}$ – variable costs for 1 hour of movement when moving cargo.

The specified cost elements are selected from known studies (Akyüz et al., 2023).

Since the variable mileage costs depend on the average operating speed of the vehicle, the following values were adopted for the respective types of transport: heavy-duty highway vehicles - 60 km/h, medium-duty vehicles - 66 km/h; railway trains (32 carriages) – 30 km/h; river barges - 12 km/h. (Akyüz et al., 2023, Lee and Choo, 2016). According to these assumptions, the actual distances between transport nodes, which are measured on maps, were converted into average travel times $t_{ij}$.

The algorithm consists of seven steps that are repeated periodically (Fig. 3). Check for the presence of closed cycles in the initial graph $G(Q,U)$ during initialization, as well as its derivative graph $H(E,Y)$, which is written in the form of matrices ($t_{ij}$), ($k_{ij}$), ($\theta_{ij}$), ($f_{ij}$), ($\tau_{ij}$), ($\tau_{ij}^{max}$) before performing calculations. If such cycles are detected, it is necessary to analyze all the arcs included in the cycle and eliminate the conflicting arcs. Also specify the total volume of transportation $K_{E}$, which must be performed on this multimodal route. To study the influence of transportation volumes on the optimal structure of a multimodal chain, it is necessary to set the range of change of $K_{E}$, i.e. $[K_{E}^{min} \ldots K_{E}^{max}]$ based on market-requests. It is necessary to determine the period, during which $K_{E}$ of goods must be transported when determining the given range. The first step is that it
is necessary to construct all possible combinations of routes in the form of an ELO sequence without repetitions.

The number of ELOs in the route, except for $e_1, e_N$, is not known in advance, although the ELOs are selected from the previously known set $\{e_2, e_{N-1}\}$.

Fig. 3. Block diagram of the algorithm for calculating parameters of multimodal routes
The easiest way to organize such a combination is to use nested loops, in which each loop will go through all the elements of a given set sequentially. In the first step, determine all possible paths in the graph G based on the recurrence relation:

$$X_{i,j} = T(X_{i,j} - 1)_{\text{max}},$$  \hspace{1cm} (6)

where $X_{i,j}$ is a vector of variables, each of them get a value $x_{i,j} = \Psi$, $\Psi = 1, 2, \ldots \Psi$, which means that arc $x_{i,j}$ of graph $G$ is used by routes. The recursive procedure for finding all possible routes that run from $q_1$ to $q_N$ is developed on the basis of (6). Recursion is based on searching with nested loops (Chen, Q., and Chen, H., 2013). If the vertices $q_i$, $q_j$ are included in the $s$-th route found, they should be included in the set $M_s$, $s=1..S$. The following steps 2-7 are repeated for every $s$-th route found. Determine the minimum tact for each section of the route in the second step using the expression:

$$\tau_{i,j}^{\text{min}} = \frac{t_{i,j}}{f_{i,j}^{\text{max}}},$$  \hspace{1cm} (7)

where $f_{i,j}^{\text{max}}$ is the maximum actual number of vehicles that can be used on the route section $q_i$-$q_j$.

The third step. Determine the maximum throughput of each section of the route using the expression:

$$\mu_{i,j}^{\text{max}} = \frac{k_{i,j}}{\tau_{i,j}^{\text{min}}}.$$  \hspace{1cm} (8)

Among all sections of the route, determine the one, for which $\mu_{i,j}^{\text{max}}$ is minimal. This section will be the bottleneck of the route, and relative to it, the parameters of all other sections are selected so that the condition $\mu_{i,j}^{\text{const}}$ is fulfilled.

In the fourth step, we adjust the actual carrying capacity of vehicles of each section of the route, except for the one for which $\min\{\mu_{i,j}^{\text{max}}\}$, so as to ensure the condition of constancy of the average intensity. Let’s introduce the notation: $\mu_{\text{min}} = \min\{\mu_{i,j}^{\text{max}}\}$ – the throughput of a multimodal logistics chain. Then the adjustment is performed according to the expression:

$$\frac{k_{i,j}'}{\tau_{i,j}} \approx \mu_{\text{min}},$$  \hspace{1cm} (9)

where $k_{i,j}'$ is corrected actual vehicle carrying capacity on the site, $k_{i,j}' < k_{i,j}$. Adjustment can be made only to the side of decrease $k_{i,j}$.

Determine the actual number of vehicles required by expression (4) and the number of cycles that need to be performed in order to transport goods on the site in the fifth step:

$$n_{c,i,j} = \frac{K_S}{k_{i,j}}$$  \hspace{1cm} (10)

Determine the section among all sections of the route where the number of cycles is minimal $n_{c}^{\text{min}}$.

In the sixth step, determine the indicators of the multimodal route:

- the total duration of movement of vehicles with cargo:

$$T_S = \sum_{i=1}^{N} \sum_{j=2}^{N} n_{c,i,j} \cdot t_{i,j}, \text{ } i, j \in M_S,$$  \hspace{1cm} (11)

- the maximum possible duration of the project:

$$T \sum_{i=1}^{N} \sum_{j=2}^{N} t_{i,j} + \tau_{\xi,N} \cdot (n_{c,\xi,N} - 1)_{\text{max}} < \tau_{\xi,N} \in M_x,$$  \hspace{1cm} (12)

where $M_x$ is the set of vertices of the graph $H$ belonging to the route $x$ under consideration; $\tau_{\xi,N}$ is the ELO tact, which is performed last in the multimodal route chain; $n_{c,\xi,N}$ is the number of execution cycles of the last in the ELO chain.

We calculate the costs per unit of the moved product according to the expression:

$$c_k = \left(\sum_{i=1}^{N} \sum_{j=2}^{N} (C_{cr} + C_{tr} \cdot t_{i,j} \cdot n_{c,i,j} + C_{tr} \cdot T_{\text{max}}) \cdot \frac{1}{K_S}\right).$$  \hspace{1cm} (13)

As it can be seen, the cost of funds depends, first of all, on the type of transport used. According to the data given in the works (Akyüz et al., 2023, Van Riessen et al., 2015), the components of these costs are:

- for automobile heavy-duty transport: $C_{c.r} = € 76.5$; $C_{t.r} = € 0.58$; $C_{c.t.r} = € 1.36$;

- for automobile medium-load transport: $C_{c.r} = € 44.8$; $C_{t.r} = € 0.28$; $C_{c.t.r} = € 0.45$;

- for railway transport in closed wagons of 66 tons:
C\textsubscript{c.r} = €153.0; C\textsubscript{l.r} = €0.037; C\textsubscript{t.r} = €0.048;
charge for river barges:
C\textsubscript{c.r} = €0; C\textsubscript{l.r} = €0.082; C\textsubscript{t.r} = €3.2.
We will show calculation examples for two arbitrary routes \(M_1, M_2\).
Let's assume that the volume of transportation for all examples is the same and is \(K_2=170\) cargo units. The first example concerns the
all-motor route \(M_1\), which consists of three sections 1–2–3–4. Trucks of the same load \(k_{ij}^\text{max}=34\)
packages can be used on each site. The maximum number of involved trucks can be, respectively, \(f_{1,2}^\text{max}=3, f_{2,3}^\text{max}=5, f_{3,4}^\text{max}=4\).
The duration of the movement of trucks along the sections of the route, together with the preparatory and final time is \(t_{1,2}=7.5\) hours, \(t_{2,3}=8.5\) hours, \(t_{3,4}=14\) hours.
There are no time window restrictions on sections of the route. The minimum tact for each section of the
route, according to (7) is \(\tau_{1,2}^\text{min}=\frac{7.5}{5}\) hours, \(\tau_{2,3}^\text{min}=\frac{8.5}{5}\) hours, \(\tau_{3,4}^\text{min}=\frac{14}{5}\) hours. Checking the capacity of the
route sections: \(\mu_{1,2}^\text{max}=\frac{7.5}{2.5}, \mu_{2,3}^\text{max}=\frac{8.5}{1.7}, \mu_{2,3}^\text{max}=\frac{14}{3.5}\).
The minimum throughput of the third section is 9.7 packets per hour.
The other two areas are adjusted for carrying capacity, reducing the actual load of vehicles. So, on section 1-2, with an actual load of 25 packages, the throughput will be \(\mu_{1,2}=25/2.5=10 \approx 9.7\) packets/hour.
In section 2-3 at \(k_{2,3}=17\) \(\mu_{2,3}=17/1.7=10 \approx 9.7\) packets/hour. The carrying capacities of the sections are
equalized in case of underloading of vehicles, or the use of vehicles with a lower load capacity. The actual number of vehicles per section of the route.
Section of the route 1-2: \(f_{1,2}=\frac{7.5}{2.5}=3\);
Section of the route 2-3: \(f_{2,3}=\frac{8.5}{1.7}=5\);
Section of the route 3-4: \(f_{3,4}=\frac{14}{3.5}=4\).
Since the tact of the transportation in the areas has not changed, the maximum specified number of vehicles was used. The number of transport cycles in
sections, according to formula (10) is: 1-2–7, 2-3–10, 3-4–5.
The total duration of movement of vehicles with cargo:
\(T_F = 7.5 \cdot 7 + 8.5 \cdot 10 + 14 \cdot 5 = 207.5\) hours.
The maximum possible duration of the project:
\(T_{\text{max}} = 7.5 + 8.5 + 14 + 3.5(5-1) = 440\) hours.

The specific costs of transporting a unit of cargo on a given multimodal route are calculated as follows.
We take into account the costs of heavy-duty transport. Due to the fact that on the second section the actual load of vehicles is reduced to 50% of
heavy-duty vehicles, \(k_{2,3}=17\), it is advisable to use medium-duty vehicles and choose the appropriate structured costs for them.

\[
c_k = \left( \frac{(76.5 + 0.58 \cdot 7.5 \cdot 7 + 1.36 \cdot 440) + (44.8 + 0.28 \cdot 8.5 \cdot 10 + 1.36 \cdot 440) + (76.5 + 0.58 \cdot 14 \cdot 5 + 1.36 \cdot 440)}{170} \right)
\]
\[
= 12.28\, \text{€/cargo unit}.
\]

The coordination of the work of trucks on the sections of the route is shown in the cyclogram of fig. 4.
As it can be seen from Fig. 4, the ELO of the multimodal delivery process consists of cycles that are subject to the tact. Tacts of different ELOs may differ, but ELO synchronization is achieved by changing the number of cargo units band. If one makes a cross-section of the cyclogram in any period it can be noticed that the estimated number of vehicles that are on the route section at the same time, i.e. the front, is preserved. It can also be seen that the organizational downtimes, which are displayed as gaps between segments, are minimal.

Another variant of the \(M_2\) route, which is taken for comparison, is the road-water route. Sequence of route operations 1-3-7. The duration of the movement of vehicles on the sections of the route: \(t_{1,3} = 9\) hours, \(t_{3,7} = 29\) hours. The schedule of movement of a river barge depends on the level of its loading. The number of vehicles (trucks with a capacity of 34) is \(f_{1,2} = 2\). Number of water barges \(f_{3,7} = 1\). The cargo capacity of the water barge is more than 600 transport packages, so we believe that \(k_{3,7}^\text{max} =K_2=170\).

Let's determine the following parameters of the route after completing all steps of the algorithm.
Minimal tacts: \(\tau_{1,3}^\text{min}=\frac{9}{9}\) hours, \(\tau_{3,7}^\text{min}=\frac{29}{9}\) hours.
Capacity of route sections: \(\mu_{1,3}^\text{max}, \mu_{3,7}^\text{max}\).
River transport is the bottleneck of this route. As a result of adjusting the carrying capacity of trucks, it was determined that $k_{1.3}=26$, which equalizes the carrying capacity of the sections. The required number of vehicles on the section 1-3 coincides with the given one. The number of cycles in section 1-3 is 7. The total duration of the project along the route 1-3-7, calculated by formula (12) is $T_{\text{max}}=146$ hours. The duration of cargo transportation by all modes of transport is $T_\Sigma = 9 \cdot 7 + 29 \cdot 1 = 92$ hours.

Fig. 4 Cyclogram of the multimodal route $M_1$

$T_{\text{max}}=440$ hours

Fig. 5. Cyclogram of the $M_2$ route at $K_\Sigma=170$ total units of cargo

$T_{\text{max}}=146$ hours
4. Computational experiments

The developed analysis and optimization methodology was applied to the transnational multimodal route, which is being designed (Poland-Ukraine set up ‘Black Sea to Baltic’). The transnational multimodal route should run from the Turkish seaport of Karas in the Black Sea to the seaport of Chornomorsk, near Odessa (Ukraine). Next, the mainland part of the route will run through the territory of Ukraine and Poland to the seaport of Gdansk in Poland (Fig. 6).

Actually, the continental part of the route is multivariate. It can consist of several sections and be carried out by three types of transport: by road (by heavy-duty road trains with a capacity of 34 standard transport packages, or 2 TEU), by rail (with a capacity of 66 TEU), or by river dry cargo feeder barge (with a capacity of 4,000 tons or about 300 TEU). At the moment, it has not been decided whether this transport corridor will be intermodal.

Let’s consider the problem of substantiating the route structure, taking into account the possibility of dividing cargo delivery groups, without loss of generality. In fig. 6 presents an oriented graph, the vertices of which are the possible times of arrival of the corresponding type of transport at the nodal point of the transport network.

To determine the duration of transportation on the mainland part of the route, one needs to lay out all possible paths in the graph from the top $e_1$ (Chornomorsk) to the top $e_{10}$ (Gdańsk). There are no cycles and loops in the graph due to the fact that all connections between the vertices of the graph were analyzed for the rational expediency of performing the corresponding ELO.

![Diagram](image-url)

Fig. 6. Scheme of possible routes for cargo delivery along the transport corridor between sea terminals: Karas (Turkey) – Chornomorsk (Ukraine) – Gdansk (Poland)
So, for example, although there is a road from Chornomorsk through Kyiv to Gdańsk, it was not taken into account, because its length, the transport conditions for making a trip on it are a priori unacceptable in terms of duration, delivery delays, working time of drivers and the required number of trucks. All other arcs that reflect communication paths, can ensure high-quality and uninterrupted delivery in the required direction when included in a combined delivery route. These arcs have an estimate of the average duration of movement of the respective vehicles, taking into account statistical data. Individual transport points are multimodal nodes, as they contain transshipment docks and stations of various types of transport (Chornomorsk, Kherson, Kyiv, L’viv, Warsaw). They also provide for short-term storage in the form of intermediate warehouses and/or container sites. Other transport points are transit, but they also have the possibility of transshipment to another transport of the same type. This is especially important for road delivery, as it greatly simplifies the construction of the permissible transport cycle of trucks and reduces their downtime and idle mileage. The availability of the maximum number of vehicles on this or that section of the network was determined based on the transportation capacity of the transport and logistics company.

5. Discussion of results
When applying the algorithm to this multimodal scheme of transportation along the route Chornomorsk-Gdańsk, the range of changes in the total cargo flow was set within the limits of 100-5000 transport packages per one working hour of the terminals. If we take into account the average weight and dimensions of one standard transport package, then in the equivalent it turns out to be 0.036 .. 1.15 million TEU per year. The number of 1.15 million TEU corresponds to the maximum throughput capacity of the Chornomorsk seaport as of 2016. Group party transportation was considered. The multimodal scheme is designed for the transportation of goods that are sent in a large group, such that a single shipment from the initial continental point of the route is significantly smaller than the size of the entire order group, that is, \( k_{1,j} < K_2 \). Reloading and transportation of adjacent shipments are not allowed along the route. At the nodal points of the route, transshipment to another vehicle may be carried out in order to reduce the cost price/reduce the duration of delivery of the entire group of goods. The total duration of the entire project (transportation of the entire amount of cargo from the port of Karas to the port of Gdańsk via the port of Chornomorsk) is not standardized, except for those cases when, the duration of delivery is limited by the maximum allowable time fund of vehicles on certain sections of the route. The task is to choose the routes of multimodal cargo transportation that are optimal in terms of specific costs for the movement of one package with a given total volume of transportation \( K_2 \). To perform such a task, the developed methodology and algorithm for calculating the dependence of \( c_k(K_2) \), \( T_{\max}(K_2) \), \( T_2(K_2) \) were applied. After performing the 1st step of the algorithm, 28 possible variants of multimodal routes were found, which are shown in Appendix. The resulting routes can be divided into four groups according to their composition: 1) purely automobile; 2) road and railway; 3) rail and water; 4) railway, water and road. After the execution of the calculation schedule, \( c_k(K_2) \) dependencies were obtained for all 28 routes (Fig. 7). For all routes, the dependences \( c_k(K_2) \) are piecewise continuous with continuity on the left. Each continuous section means one, qualitatively different transport and technological scheme. For example, when organizing the delivery of goods by Route 25, which is as follows (see Fig. 6): 1–5 (railway) 5-9 (road) 9-10 (road), with the volume of transportation 1170..1540 packages of transportation, costs per package is €1/package. And with \( K_2=1540..2220 \) the costs are €0.9/package. The dependencies of those routes in fig. 7 have the lowest costs among all routes over the entire range of \( K_2 \). If for some route the costs are higher for the entire range of transportation volumes, then such routes are excluded from consideration. However, it can be noted that for some routes the dependences \( c_k(K_2) \) intersect. Fig. 7 shows that, for example, routes 21 and 25, 23 and 26, 21 and 26 have intersection points. The presence of such intersections of the dependences \( c_{k,i}(K_2) \) and \( c_{k,i}(K_2) \) at the point \( K_{2,1} \) means that the route, which costs are lower before \( K_{2,1} \) has an advantage in the choice. The route will
lose ground in the competition after the growth of volumes above $K_{\Sigma}$. for a given volume of transportation. This gives grounds for a reasonable choice of routes according to the criterion of specific costs (10). Fig. 7 also shows that some routes, for example, route 15 (items 1-3-5-10) cannot be applied for the entire proposed range of $K_{\Sigma}$. This can be explained by the lack of transport capacity of vehicles on this route. Because of this, the mentioned route 15 (completely by road) can be used only for the transportation of goods with a volume of up to 540 packages. Otherwise, it will be necessary to use intermediate storage, which will significantly increase the duration and cost of delivery. By comparing the received routes by the cost of delivery over the entire range of $K_{\Sigma}$, we can distinguish three conditional zones of route profitability. The first zone $K_{\Sigma} <300$ packets is a zone of high sensitivity. Each additional 10 packages in the volume of transportation reduces specific costs by €0.1. Water transportation here is inferior to rail and even automobile transportation in terms of costs. If we also take into account the environmental friendliness of transportation preference should be given to rail transportation.

The second zone is the range of $K_{\Sigma}=300..1200$ packets, where various routes by type of transport can compete by costs. The choice of routes in this area is very large and additional criteria should be applied to make an informed decision. The third zone is a zone of large volumes of transportation, where transport and technological schemes operate almost stably, and the choice of a multimodal transportation scheme is almost obvious. For the specified example of delivery, the best in terms of minimum costs are rail-water and road-water routes, for which transportation costs per package almost do not change with increasing volumes.

Fig. 7. Dependence of the total specific costs for the movement of a unit of cargo by various multimodal routes on the total volume of cargo flow on the Chornomorsk (Ukraine) - Gdansk (Poland) route
If we compare the received routes by the maximum duration of delivery of the entire volume (Fig. 8), then the priorities of choosing routes are fundamentally different here. Road routes have an obvious advantage over the entire permissible range of $K_\Sigma$. However, their range is quite short. Road and railway routes are predominant in the zone of high sensitivity. Water and water-rail routes have an advantage in the stability zone, while rail-road delivery times are increasing rapidly.

The received routes can be similarly characterized by the indicator of the total transportation time (Fig. 9). However, purely water routes have a total advantage in terms of this indicator over the entire range of possible volumes of transportation. This is explained by the large volumes of loading of barges, ferries, and ships. At the same time, large time windows of water transport are leveled by consolidated cargoes.

As can be seen from the dependence, purely automobile routes do not have an advantage over combined ones, because in order to avoid traffic idling, trucks are often underloaded to the nominal value.

The increase in the volume of transportation on combined rail-road routes or rail routes requires the use of wagon shipments, as a result of which the duration of transportation of the entire volume of cargo increases almost linearly.

6. Conclusion

The obtained research results are distinguished by new principles of analysis of alternatives and synthesis of optimal transport and technological schemes, which were previously not taken into account by researchers. These principles have matured due to the fact that the volumes of goods delivered in multimodal communication have acquired very wide intervals of numerical values. Freight carriers try to satisfy the most diverse demands of consumers. However, due to the limitations of modern transport technologies, there are permissible values of the duration of delivery, or parts of delivery, of the minimum or maximum size of a group of goods. Hence the urgent need to consider time windows, process timing and synchronization, bulkiness of cargo and a new understanding of the transport package.

![Fig. 8. Dependence of the maximum duration of transportation by various multimodal routes between Chornomorsk (Ukraine) – Gdansk (Poland) on the total volume of cargo units](image-url)
When considering the model of multimodal freight transportation, it is necessary to accept it as a discrete flow characterized by time windows. Such an indicator as the tact of a discrete cyclic process can be used in this case as an argument for substantiating interoperational interaction at individual stages of the transportation process. Another important modeling principle to consider is process continuity. Due to the fact that multimodal cargo transportation is carried out on very long routes, time is one of the most important criteria for their optimization. The sum of the durations of individual stages of a multimodal process is not equal to the duration of the process as a whole.

Glossary

Elementary Logistics Operation is qualitative transformation of the material flow, which leads to one of the possible changes in cargo flows occur at fixed moments of time, and the flow itself has the properties of accelerating / decelerating, splitting or merging.

Tact is the duration of separate division of discrete material flow, which limits its duration.

Cycle is a period of time between successive qualitative changes of the same material flow.

Cargo group is a set of cargo units that are in one ELO.

Front of vehicles is the number of vehicles simultaneously are transporting cargo during the execution of the one ELO.

Conflicting arcs these are arcs that cannot simultaneously represent the same transport process due to the presence of ambiguity in its schedule.

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References


Appendix Table A. Routes Details
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