DEVELOPMENT OF A MATHEMATICAL MODEL OF THE GENERALIZED DIAGNOSTIC INDICATOR ON THE BASIS OF FULL FACTORIAL EXPERIMENT

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Abstract: Purpose. The aim of this work is to develop a mathematical model of the generalized diagnostic indicator of the technical state of traction substations electrical equipment. Methodology. The main tenets of the experiment planning theory, methods of structural-functional and multi-factor analysis, methods of mathematical and numerical modeling have been used to solve the set tasks. Results. To obtain the mathematical model of the generalized diagnostic indicator, a full factorial experiment for DC circuit breaker have been conducted. The plan of the experiment and factors affecting the change of the unit technical condition have been selected. The regression equation in variables coded values and the polynomial mathematical model of the generalized diagnostic indicator of the circuit breaker technical condition have been obtained. On the basis of regression equation analysis the character of influence of circuit breaker diagnostic indicators values on generalized diagnostic indicator changes has been defined. As a result of repeated performances of the full factorial experiment the mathematical models for other types of traction substations power equipment have been obtained. Originality. An improved theoretical approach to the construction of generalized diagnostic indicators mathematical models for main types of traction substations electric equipment with using the methods of experiments planning theory has been suggested. Practical value. The obtained polynomial mathematical models of the generalized diagnostic indicator D can be used for constructing the automated system of monitoring and forecasting of the traction substations equipment technical condition, which allows improving the performance of processing the diagnostic information and ensuring the accuracy of the diagnosis. Analysing and forecasting the electrical equipment technical condition with the using of mathematical models of generalized diagnostic indicator changes process allows constructing the optimal strategy of maintenance and repair based on the actual technical condition of the electrical equipment. This will reduce material and financial costs of maintenance and repair work as well as the equipment downtime caused by planned inspections and repair improving reliability and uptime of electrical equipment.

Key words: electricity, traction substation, maintenance, diagnostics, full factorial experiment, mathematical model, regression equation.

1. Introduction

The efficiency and reliability of traction substations electrical equipment (EE) depends on its technical condition. Modern EE has a fairly high estimates of reliability (Szeląg, A., 2017), however, in the process of operation under the influence of external conditions and variable modes of operation the initial state of the equipment is continuously deteriorating, which leads to additional energy losses, reduction of operational reliability and the growth rate of equipment and the number of failures. The reliability of EE during the life cycle depends not only on the quality of manufacture, but also on the operating conditions and quality of maintenance and repair. The providing an inspection, verification, regulation and monitoring of the technical condition of EE and using an innovative technologies of diagnostics are the basis of the strategy of improving the technical object operational reliability (Matusevych, O. O. & Mironov, D. V., 2015; Matusevych, O. O. & Sychenko, V. G. & Bialon A., 2016). A new direction in development of technical
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maintenance and repair system is the development of approaches based on individual observation and prediction of the real change of the equipment technical condition during operation. It is necessary to develop a means of obtaining diagnostic information as well as mathematical methods and models that take into account the main factors influencing the technical condition of EE (Szubartowski, M., 2013).

As is known, for assessment of technical state the mechanical (vibration), thermal, electrical and other factors are analyzed. These factors have different physico-chemical nature and lead to EE individual properties changes. In this case, the assessment of technical condition of electrical equipment individual properties runs more or less satisfactorily. However, the overall assessment of the equipment technical condition is extremely difficult due to the need comparing the different physical nature factors and the absence the relationship between them, which can be described by the analytical equation. Therefore, it is proved that for adequate evaluation of the quality of power equipments operational indicators it is necessary using the generalized characteristics of their work (Mironov, D. V., 2015).

Knowledge of the equipment generalized technical condition allows assessing the reliability of the whole technological complex. Using a generalized diagnostic indicator as a basic EE technical condition evaluation allows to increasing the accuracy of determining the actual equipment technical condition. However, automating the process of EE diagnosis involves the using of mathematical models of the generalized diagnostic indicator of the equipment technical state to improving the performance of the process and ensuring the reliability of the diagnosis. The task of modeling and forecasting changes in EE technical state is multifactorial and nonlinear, which leads to the application of various methods of its solution. One way of solving this problem is the use of mathematical methods of the experiment planning theory.


Factors that are used in the planning of the experiment should satisfy the following requirements (Adler, Yu. P. & Markova, E. V. & Granovskiy, Yu. V., 1976):
1) Controllability.
2) Operationally.
3) Measurement accuracy.
4) Uniqueness.

During the planning of the experiment the several factors are changed at the same time. Therefore it is very important to formulate the requirements, which apply to a combination of factors. First of all the requirement of compatibility is determined. Factors compatibility means that all of their combinations are feasible and safe. During the planning of the experiment the factors independence is very important, i.e., the possibility of establishing a factor at any level, regardless of other factors levels.

The next step is to define the primary level and range of factors change and their variation on two levels. In this case, if the number of factors is known, we can immediately find the number of experiments required to implement all possible combinations of factors levels:

\[ N = 2^k \]  

(1)

where:
- \( N \) is the number of experiments,
- \( k \) is a number of factors,
- 2 is the number of levels.

To simplify the notation of the experiment conditions and processing the experimental data, the scales on the axes are selected so that the upper level corresponds to +1, lower to -1, and the main to zero. For factors with a continuous determination region, this can always be done using the transform

\[ x_j = \frac{\bar{x}_j - \bar{x}_{j0}}{I_j} \]

(2)

where:
- \( x_j \) is the coded value of the factor,
- \( \bar{x}_j \) is the natural value of the factor,
- \( \bar{x}_{j0} \) is the natural value of the basic level,
- $I_j$ is the range of variation,
- $j$ is the number of the factor.

For qualitative factors with two levels, one level is denoted by +1 and the other is -1. The order of the levels doesn’t matter.

Simplicity and adequacy are the main requirements in determining the form of mathematical models. It is believed that algebraic polynomials are the simplest models. Polynomials are linear relative to the unknown parameters of the model, which greatly simplifies the processing of the experiment results (Adler, Yu. P. & Markova, E. V. & Granovskiy, Yu. V., 1976). Therefore, as a mathematical model the algebraic polynomials has been chosen in this work. During the plan selection the optimality criteria and the number of experiments primarily is taken into account (Gard, M. & Levinson, S. J. & Ferraro, S. B. & Jimenez, J. A., 2012; Mills, K. L. & Filliben, J. J. & Haines, A. L., 2015). In our case, it is clear that the required plan should be two-level (because we are interested in linear model), orthogonal and rotatably. Orthogonality allows to move along the gradient is proportional to the coefficients of the linear model and independently to interpret the effects. Rotatability provides assured equality of prediction variances with the motion in any direction from the center of the experiment. All these requirements are satisfied by a full factor experiment type $2^n$ (Nalimov, V. V., 1965).

The matrix of the experiment planning must satisfy the following requirements:

1) The symmetry relative to the center of the experiment

$$\sum_{i=1}^{N} x_{ji} = 0, \quad (3)$$

where:
- $j$ is the number of the factor,
- $N$ is the number of experiments,
- $j = 1, 2, ... k$.

2) The normalization condition

$$\sum_{i=1}^{N} x_{ji}^2 = N. \quad (4)$$

This is a consequence of the fact that the values of the factors in the matrix are set to +1 and -1.

3) Orthogonality of the planning matrix

$$\sum_{i=1}^{N} x_{ji} \cdot x_{ju} = 0, \quad j \neq u, \quad j, u = 0, 1, 2, ... , k. \quad (5)$$

4) Rotatability. Point in the planning matrix are chosen so that the accuracy of the optimization parameter values prediction is the same at equal distances from the center of the experiment and does not depend on the direction.

The results of the experiment depending on the setting of factors at a given level are recorded in the last column of the matrix of full factorial experiment. Then the results of the experiment are processed:

1) The calculation of the reproducibility variance.

$$s^2_{(y)} = \frac{1}{N-1} \sum_{i=1}^{N} (y_i - \bar{y})^2, \quad (6)$$

where:
- $\bar{y}$ is the expected value of the response function,
- $N - 1$ is the number of freedom levels.

2) Determination of regression coefficients.

$$b_j = \frac{\sum_{i=1}^{N} y_i \cdot x_{ji}}{N}, \quad b_0 = \bar{y}. \quad (7)$$

3) Testing the significance of regression coefficients.

$$s^2_{(b_j)} = \frac{s^2_{(y)}}{N}, \quad \Delta b_j = \pm t \cdot s_{(b_j)}, \quad (8)$$

where:
- $t$ is the table value of Student’s criterion with the number of freedom levels, which was determined by $s^2_{(y)}$ and the chosen significance level (in our case for 0.05).

The coefficient is significant if its absolute value is greater than or equal to $\Delta b_j$. 

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4) Checking adequacy of the model.

\[ F = \frac{s_{ad}^2}{\hat{s}_{y1}^2}, \]  \hspace{1cm} (9)

where:
- \( F \) is Fisher's criterion. In this case \( F_{calc} < F_{tabl} \); 
- \( f \) is the number of freedom levels, \( f = N - (k + 1) \); 
- \( s_{ad}^2 \) is the dispersion of adequacy:

\[ s_{ad}^2 = \frac{\sum_{i=1}^{N} (y_i - \hat{y}_i)^2}{f}, \]  \hspace{1cm} (10)

where:
- \( \hat{y}_i \) is the response function calculated by the regression equation;
- \( y_i \) is the response function obtained in the experiment.

The result of the full factorial experiment is written as a polynomial regression equation (Alzoubi, K. & Lu, S. & Sammakia, B. & Polikis M., 2011). The last step in the experiment is the interpretation of results and decision making after constructing a model. The task of interpreting is the fairly complex. The degree of influence of each factor on the optimization parameter is set (Chen, F. & Ma, X. & Zhao, Y. & Zou, J., 2011). In some tasks the construction of the regression equation for the natural values of the factors is required. The equation for the natural variables can be obtained using the transition formula (2). This eliminates the interpretation possibility and the influence of factors on the values and signs of the regression coefficients. However, the resulting equation can be used to predict the optimization criterion changes.

The linear model is inadequate, to obtain adequate models the following methods are used: changing the intervals of factors variation, the center plan transfer, the plan completion (Bondar', A. G. & Statyukha, G. A. & Potyazhenko, I. A., 1980; Asaturjan, V. I., 1983). The latter method is associated with the transition to the orthogonal central composite experiment plan.

2. The construction of a mathematical model

To obtain the mathematical model of the generalized diagnostic indicator, we have conducted a full factorial experiment for DC circuit breaker in this work. A generalized diagnostic indicator \( D(t) \) has been used as the response function (Mironov, D. V., 2015). According to (The Ministry Of Infrastructure, 2008; Kuznecov, V. G. & Galkin, O. G. & Efimov, O. V. & Matusevych, O. O., 2009) the main diagnostic indicators and their changes limits for obtaining \( D \) have been identified (table 1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>The main level</th>
<th>The upper limit</th>
<th>The lower limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main contacts press, kgp</td>
<td>30</td>
<td>33</td>
<td>27</td>
</tr>
<tr>
<td>The spring tension, kgp</td>
<td>35</td>
<td>38</td>
<td>32</td>
</tr>
<tr>
<td>The number of outages, th.</td>
<td>40</td>
<td>60</td>
<td>20</td>
</tr>
<tr>
<td>The area of adjoining the main contacts, %</td>
<td>85</td>
<td>94</td>
<td>76</td>
</tr>
<tr>
<td>The failure of the main contact, mm</td>
<td>1.125</td>
<td>1.515</td>
<td>0.735</td>
</tr>
</tbody>
</table>

Considering the optimality criteria and the number of factors we have selected the full factorial experiment plan type \( 2^5 \) of first order (Nalimov, V. V. & Golikova, T. I., 1980) and composed the full factorial experiment planning matrix (table 2). According to the expression (1) the required number of experiments has been defined \( (N = 32) \). In this matrix the values of the factors is used in all possible combinations. The first column corresponds to the coefficients of the model constant term, columns from 2nd to 6th correspond to the factors value and the 7-th column is the value of the system response.

<table>
<thead>
<tr>
<th>The experiment number</th>
<th>x0</th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>34</td>
<td>43</td>
<td>40</td>
<td>97</td>
<td>2.5</td>
<td>0.638</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>34</td>
<td>47</td>
<td>40</td>
<td>97</td>
<td>2.5</td>
<td>0.601</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>38</td>
<td>43</td>
<td>40</td>
<td>97</td>
<td>2.5</td>
<td>0.587</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>38</td>
<td>47</td>
<td>40</td>
<td>97</td>
<td>2.5</td>
<td>0.553</td>
</tr>
</tbody>
</table>
To simplify the experiment and processing the experimental data, the transformation from the factors natural values to the coded values has been performed by the expression (2). The planning matrix of the full factorial experiment with the columns of the factors interaction (columns 7 – 32) has been completed (table 3).

The constructed matrix satisfies the requirements of (3) – (5). We have written the model of the generalized diagnostic indicator in the form of the regression equation:

\[
D = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + b_3 \cdot x_3 + b_4 \cdot x_4 + \\
b_5 \cdot x_5 + b_6 \cdot x_1 \cdot x_2 + b_7 \cdot x_1 \cdot x_3 + b_8 \cdot x_1 \cdot x_4 + \\
b_9 \cdot x_1 \cdot x_5 + b_{10} \cdot x_2 \cdot x_3 + b_{11} \cdot x_2 \cdot x_4 + b_{12} \cdot x_2 \cdot x_5 + \\
b_{13} \cdot x_3 \cdot x_4 + b_{14} \cdot x_3 \cdot x_5 + b_{15} \cdot x_4 \cdot x_5 + \\
b_{16} \cdot x_1 \cdot x_2 \cdot x_3 + b_{17} \cdot x_1 \cdot x_2 \cdot x_4 + b_{18} \cdot x_1 \cdot x_2 \cdot x_5 + \\
b_{19} \cdot x_1 \cdot x_3 \cdot x_4 + b_{20} \cdot x_1 \cdot x_3 \cdot x_5 + b_{21} \cdot x_1 \cdot x_4 \cdot x_5 + \\
b_{22} \cdot x_2 \cdot x_3 \cdot x_4 + b_{23} \cdot x_2 \cdot x_3 \cdot x_5 + b_{24} \cdot x_2 \cdot x_4 \cdot x_5 + \\
b_{25} \cdot x_3 \cdot x_4 \cdot x_5 + b_{26} \cdot x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5 + \\
b_{27} \cdot x_1 \cdot x_2 \cdot x_3 \cdot x_4 + b_{28} \cdot x_1 \cdot x_2 \cdot x_3 \cdot x_5 + \\
b_{29} \cdot x_1 \cdot x_2 \cdot x_4 \cdot x_5 + b_{30} \cdot x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5.
\]

(11)

The regression coefficients by the formula (7) have been determined. The statistically significant regression coefficients by the formula (8) have been chosen. As a result of the full factorial experiment the regression equation has been obtained:

\[
D = 0.5583 + 0.02649 \cdot x_1 + 0.02868 \cdot x_2 - 0.02014 \cdot x_3 + 0.02566 \cdot x_4 + 0.02497 \cdot x_5
\]

(12)

The resulting model is checked for adequacy by the Fisher test according to the formula (9). The calculated value of Fisher criterion is \( F_{calc} = 2.1 \). The table value of the Fisher criterion is \( F_{tabl} = 1.8874 \) (Adler, Yu. P. & Markova, E. V. & Granovskiy, Yu. V., 1976). Because the condition \( F_{calc} < F_{tabl} \) is not met, the model (12) can not be considered adequate. Considering the obtained result it was decided to finish the plan of full factorial experiment to the orthogonal central compositional plan of second order without loss of information about previous experiences (table 4).

### Table 3. A fragment of a coded planning matrix of full factorial experiment

| The experiment number | x0 | x1 | x2 | x3 | x4 | x5 | x1x2 | x1x3 | x1x4 | x1x5 | x2x3 | x2x4 | x2x5 | x3x4 | x3x5 | x4x5 | x1x2x3 | x1x2x4 | x1x2x5 | x1x3x4 | x1x3x5 | x1x4x5 | x2x3x4 | x2x3x5 | x2x4x5 | x3x4x5 | x1x2x3x4 | x1x2x3x5 | x1x2x4x5 | x1x3x4x5 | x2x3x4x5 | D |
|-----------------------|----|----|----|----|----|----|------|------|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1                     | 0  | 0  | 0  | 0  | 0  | 0  | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0.48589 |
| 2                     | 0  | 0  | 0  | 0  | 0  | 0  | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0.53429 |
| 3                     | 0  | 0  | 0  | 0  | 0  | 0  | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0.59216 |
| 4                     | 0  | 0  | 0  | 0  | 0  | 0  | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0    | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0.48589 |
After the same transformations and calculations the following regression equation has been obtained:

\[
D = 0.556036886 + 0.02709781 \cdot x_1 + 0.034245 \cdot x_2 - 0.02015008 \cdot x_3 + 0.025485 \cdot x_4 + 0.026203 \cdot x_5 + \cdots
\]

(13)

The analysis of the regression equation (13) allowed defining the character of the diagnostic factors influence on the change of DC circuit breaker technical condition. If you increase the value of factors such as the main contacts press, the spring tension, the area of adjoining the main contacts, the failure of the main contact (the regression coefficients of these factors have a positive sign) and reduce the value of the outages number (the regression coefficient of the factor has a negative sign) the value of the generic diagnostic indicator will increase. If you decrease the values of diagnostic factors such as the main contacts press, the spring tension, the area of adjoining the main contacts, the failure of the main contact and increase the value of the outages number the breaker status will be closer to pre-failure. It will signal the need to remove it from service and execute the maintenance and repair work.

We have conducted an audit of the obtained regression model and founded that the regression model is adequate \((F_{calc} = 1.535 < F_{tabl} = 1.6)\) (Adler, Yu. P. & Markova, E. V. & Granovskiy, Yu. V., 1976). The simulation results of the generalized diagnostic indicator shown in fig. 1. As follows from the analysis of Fig. 1 the value of the generic diagnostic indicator according to the results of the experiment and the regression equation are almost identical (\(s^2_{pred} = 0.002978\)). It shows the adequacy of the obtained results.
To obtain the polynomial models of the generalized diagnostic indicator for DC circuit breaker, suitable for use in prediction tasks (Voznesenskij, V. A. & Koval'chuk, A. F., 1978), it is necessary to move from coded to natural variables by the formula (2). After conversion the following expression has been got:

\[ D_{DC} = 0.347074 \cdot x_1^2 + 0.20529 \cdot x_2^2 + 0.269718 \cdot x_3^2 - 0.77776 \cdot x_4 \]

On the basis of this method we have obtained the polynomial model of the generalized diagnostic indicator for other types of traction substations power equipment, namely:

1) Oil circuit breakers \( (s_{\text{ad}}^2 = 0.002541) \)
\[
D_{oB} = 0.203348 + 0.333772 \cdot x_1 + 0.327493 \cdot x_2 + \\
+0.593883 \cdot x_3 + 0.226136 \cdot x_4 - 0.77776 \cdot x_5 + \\
+0.490774 \cdot x_1^2 - 0.38433 \cdot x_2^2 + 1.135307 \cdot x_3^2.
\]

2) Vacuum circuit breakers \( (s_{\text{ad}}^2 = 0.002254) \)
\[
D_{VB} = 0.90204 + 0.246541 \cdot x_1 + 0.2479 \cdot x_2 - \\
-0.35461 \cdot x_3 - 0.46874 \cdot x_1^2 - 0.48785 \cdot x_2^2.
\]

3) Sulfur hexafluoride circuit breakers \( (s_{\text{ad}}^2 = 0.002481) \)
\[
D_{SH} = 0.676946 + 0.31107 \cdot x_1 + 0.091242 \cdot x_2 - \\
-0.29718 \cdot x_3 - 0.31101 \cdot x_2^2.
\]

4) Power transformers \( (s_{\text{ad}}^2 = 0.002256) \)
\[
D_{PT} = -0.04269 + 0.166659 \cdot x_1 + 0.611565 \cdot x_2 + \\
+0.686812 \cdot x_3 + 0.232403 \cdot x_4 - 0.31107 \cdot x_1^2 - \\
-0.31254 \cdot x_2^2 - 0.37284 \cdot x_3^2.
\]

5) Current transformers \( (s_{\text{ad}}^2 = 0.002815) \)
\[
D_{CT} = 0.12101 + 0.139881 \cdot x_1 + 0.91763 \cdot x_2 + \\
+0.27498 \cdot x_3 - 0.34259 \cdot x_1^2 - 0.47864 \cdot x_2^2.
\]

6) Voltage transformers \( (s_{\text{ad}}^2 = 0.0025) \)
\[
D_{VT} = -0.05632 + 0.624604 \cdot x_1 + 0.310983 \cdot x_2 + \\
+0.310983 \cdot x_3 - 0.26262 \cdot x_1^2.
\]

The obtained polynomial models of the generalized diagnostic indicator may be used to evaluate and predict the technical condition of the main power equipment of DC traction substations depending on the change of diagnostic parameters. The using of these models for the calculating of the actual technical condition of EE and forecasting its changes for the automation of maintenance and
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diagnosis process for traction substations (Mironov, D. V. & Sychenko, V. G. & Matusevych, O. O., 2016) will significantly improve the performance and reliability of the EE monitoring process.

3. Conclusions
In the study it had determined that the change model of the generalized diagnostic indicator $D$ has the form of nonlinear second-order polynomial. It had proved that the using the mathematical methods of the experiment planning theory allows to building a mathematical model of generalized diagnostic indicator $D$ changes the with a sufficiently high accuracy of output result obtaining ($s_{\text{max}}^2$ for different types of traction substations power equipment varies from 0.002254 to 0.002978). After conducting a full factorial experiment, the regression equation for the traction substations power equipment and the character of the diagnostic parameters influence on the EE technical condition changes have been obtained. To predict changes in the technical condition of power EE the polynomial models in natural values of the diagnostic factors have been obtained. The obtained polynomial mathematical model of the generalized diagnostic indicator $D$ may be used to construct the automated system of monitoring and forecasting of the traction substations equipment technical condition and to improve the performance of the diagnostic information processing and ensure the diagnosis accuracy. The mathematical model of generalized diagnostic indicator can be used in developing of the maintenance and repair optimal strategy based on actual technical condition of EE. This will reduce material and financial costs of maintenance and repair works.

References


