A STUDY ON TWO-STAGE SELECTION MODEL OF TOURISM DESTINATION AT THE SCALE OF URBAN AGGLOMERATIONS

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Abstract:
Considering that the demand of tourism destination is variable on the scale of urban agglomeration, the selection process of travel destination is divided into two stages. The traditional transportation combination model based on the multinomial Logit cannot reflect this characteristic. And it is the lack of consideration of the influence of travel distribution and the dynamic transfer of passenger flow between various transport routes. Therefore, this thesis established a combination model of travel demand distribution and transportation assignment with two-stage terminal selection characteristics based on the nested Logit. Based on the analysis of tourists' trip process on the scale of urban agglomeration, a tourist flow transport network with travel destination nest structure is constructed. The generalized cost impedance function of transportation route is constructed based on the direct cost of transportation mode and the indirect cost of travel time. Based on the characteristics of two-stage destination selection of tourists, the form of travel distribution function of tourist flow is given. Through the first-order optimization conditions, it proved that the volume of travel distribution and tourism passenger transport assignment can meet the two-stage equilibrium conditions in the equilibrium state. Based on the idea of MSA algorithm, it designed the solution algorithm of the model and verified the feasibility of the model and algorithm in a simplified example. The calculation results show that the two-stage equilibrium assignment model proposed in this paper can obtain the volume of travel distribution and transportation assignment at the same time, meanwhile compared with the multinomial logit model, the nested Logit structure fully considers the attraction measure of the city destination and the scenic spot destination, which is more in line with the choice behavior of the tourists when choosing the transportation route. Thus, it provides a new comparable method for the optimal allocation of tourism passenger flow transport network resources on the scale of urban agglomeration, and can provide data support for the transportation organization plans of government decision-making departments and tourism transport enterprises.

Keywords: transportation system engineering, combination model, nested logit, travel transportation, two-stage, balanced distribution

To cite this article:

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1. Introduction
Since 2016, the Ministry of Transport, the National Tourism Administration, etc. have provided a series of guidance and policy support for the development of tourism in the whole region, which has promoted the rapid growth of tourism. The result of tourism in China is shown in Fig. 1 (Data source: 2020 statistical bulletin on cultural and tourism development of Ministry of Culture and Tourism of the People's Republic of China). However, a sudden COVID-19 spread worldwide in 2020, which caused the suspension of China's tourism industry. With the ongoing efforts of the people across the country, the epidemic was once effectively contained. Still, the spread and development of the global outbreak made the domestic epidemic repeated and then evolved into a normalized and persistent state. In the post epidemic era, many local policies in China have advocated intra-provincial travel to reduce the epidemic infection caused by people gathering and long-distance flow. Tourism demand has also been upgraded from scenic spots to the in-depth experience of tourism destinations. The tourism ecosphere built within this urban agglomeration can not only meet the needs of epidemic prevention and control but also promote the structural upgrading and integration innovation of the tourism industry to meet the needs of people for a better spiritual life in the new era. Urban agglomerations are the highest structure and organization form in the mature stage of urban development. During the period of prevention and control of epidemic normalization, it is of great significance to study the tourism transportation problems at the scale of urban agglomerations and guide the configuration of tourism passenger transportation modes within the scope of urban agglomerations in a scientific and reasonable manner.

2. Literature review
Researchers have been trying to explain the basic laws and characteristics of tourists’ spatiotemporal behavior, and have formed corresponding theoretical results. The study on temporal distribution characteristics of tourism flows mainly focuses on the statistical analysis and prediction of tourism flows. (Kim& Moosa, 2005) took direct and indirect methods to predict the tourism flows in Australia, making comparison and analysis of the advantages and disadvantages of the two methods. (Zhu et al., 2018) proposed the copula-based approach combined with econometric models is for tourism demand analysis that can be used to predict tourist arrivals. (Petrevska Biljana, 2015) underlined the importance of identifying seasonality effects over tourism development by calculating some commonly applied indicators for measuring tourism seasonality, like Gini coefficient, Seasonality Indicator and Coefficient of Variation. The research on spatial structure characteristics of tourism flows mainly focuses on the spatial agglomeration and diffusion as well as the spatial

![Fig. 1. Tourism development in China between 2011 and 2020](image-url)
behavior pattern and spatial network structure characteristics. (Jing Qin et al., 2019) explored the spatial characteristics of China’s inbound tourist flow, the spatial patterns of tourist movement, and the tourist destination cities group based on data mining techniques, including the Markov chain, a frequent-pattern-mining algorithm, and a community detection algorithm. (Koun Sugimoto et al., 2019) examined the relationship between visitor mobility and urban spatial structures through an exploratory analysis of visitors’ movements and characteristics, which were collected from surveys with global positional system (GPS) tracking technologies and questionnaires. (Pavlovich Kathryn, 2002) examined the process of tourism destination evolution and transformation and used network theory to express these dynamics, and it emphasized structural features of architectural density and centrality. (Hwayoon Seok et al., 2020) explored how the structure of international tourism has changed longitudinally using a social network analysis. Scholars value more the choice of tourism transportation within or between cities. (Qi et al., 2020) explored the relationship between destination means of transportation and travel chain based on nested Logit, and takes two nested structures to analyze the decision-making process of travelers. (Malik et al., 2019) designed objective function of tourist routine optimization by taking neural network and particle swarm optimization algorithm base on five route parameters: distance, road congestion, weather conditions, route popularity and user preference. Through smart travel card data, (Gutiérrez et al., 2020) analyzed tourists use condition of urban public transport. (Trinh et al., 2017) took binary Logit model to study tourists mode choice behavior and believed that the main factors affecting tourists choice of transportation mode on travel route include socioeconomic factors (like family income and age of interviewees) and travel characteristics (like total travel time and travel budget). (Ghader et al., 2021) introduce a multi-dimensional continuous activity scheduling choice modeling framework. It focuses on modeling the joint choice of arrival to an activity and departure from the activity. Each of the choices is modeled in continuous time using based on the continuous cross-nested logit model. (Shanmugam et al., 2021) created a model choice from specific urban communities (Coimbatore, Erode, Salem and Trichy) to Chennai with the nested logit model. (Bastariantanto et al., 2019) attempts to measure and compare the relationship between tour type and model choice using three different modelling approaches: Multinomial Logit (MNL), Nested Logit (NL) and Cross-Nested Logit (CNL).

The research on the prediction of tourism transportation mode is mainly to divide the tourism transportation mode under the premise of given OD distribution, taking traditional survey statistics data or microblog check-in data under the Internet background. It is found that most studies tend to estimate the proportion of tourists choosing a certain mode of transportation with the help of Logit stochastic utility model with the help of origin-destination distribution. However, they did not consider the influence relationship between transportation mode choice and OD distribution. Based on transportation combination model, one can get both the volumes of travel demand allocation and transportation mode distribution to reflect the transfer of dynamic passenger flow between transportation modes and tourism destinations. The traditional traffic combination model is mostly used to study congestion in multimodal urban transportation networks. (Fan et al., 2022) develop a general FP model for CMSTA problem in multimodal urban transportation networks and consider a transit system with the realistic common line problem and congestion effects. (Amirali et al., 2019) propose a combined model of auto-transit assignment based on two complementarity formulations. The model accounts for interactions between the auto and transit modes through non-separable asymmetric demand and cost functions. (Ye et al., 2021) develop a bi-modal model for the multimodal network design problem. The upper-level problem is to determine the location and capacity of the transfer infrastructure to be built simultaneously. The transfer infrastructure to be built simultaneously. The lower-level problem is the assignment of the combined trip distribution/modal split/traffic. (Liu et al., 2020) investigates a bi-modal corridor with a congested highway and a crowded transit line. CATs and CUTs are taken into account in this paper. The equilibrium traffic flow pattern and travel costs are derived. However, the traditional transportation combination model is based on the multinomial Logit model. (Sun et al., 2021) propose a logit-based multi-class ridesharing user equilibrium assignment framework formulated as a mixed complementarity problem (MCP). (Wang et al., 2020) develop a mixed behavioral equilibrium model with explicit
consideration of mode choice (MBE-MC). For the model choice, traveler, following a logit modal split, selects among three options. (Mi et al., 2015) establish the urban resident’s travel choice model in travel peak period on the condition of mixed traveling choices of cars and buses, based on the user equilibrium theory and the traffic bottleneck theory. These cannot reflect the two-stage selection behavior of tourists at the scale of urban agglomerations, that is, choosing a city as a destination and then a scenic spot of the city as a tourism destination.

Based on the above analysis, in consideration that the travel destination demand at the scale of urban agglomerations is variable in two stages and that the travel distribution and transportation mode assignment are carried out simultaneously, this paper uses the super network method (Meng et al., 2014; Zhu et al., 2012) of equilibrium theory to establish a combined equilibrium model of travel demand distribution and transportation assignment. At the same time, in order to overcome the independence (IIA) of unrelated alternatives in the multinomial Logit model (Munoz et al., 2020), this paper constructs a two-level tree-like nested structure for travel destination selection so that, given the total number of travel starting points, the destination cities and scenic spots within the city group are selected respectively. Through the established two-stage balanced assignment model, it can obtain simultaneously the OD distribution of tourist flow and the passenger flow distribution of transportation mode. This study will help reflect the influence relationship between tourist flow distribution and transportation mode at the scale of urban agglomeration and provide technical support for tourism transportation in the post-epidemic era from the perspective of the supply of tourism facilities and transportation services. The structure of this paper is as follows: in Part III, we describe the travel problems at the scale of urban agglomerations and build a super network based on two-stage terminal selection; in Part IV, the two-stage equilibrium conditions of tourism transportation at the scale of urban agglomerations are given; in Part V, a two-stage destination selection model of the combination of tourism demand distribution and transportation assignment is established; in Part VI, the solution algorithm of the model is presented; in Part VII, an example is analyzed and verified; in Part VIII, some conclusions are provided.

3. **Description of travel issues at the scale of urban agglomerations**

3.1. **Two-stage selection process of travel destination**

It is assumed that tourists will only travel within the urban agglomerations (for example, in case of epidemic control requirements or only within the urban agglomeration after tourists enter the city cluster). At the scale of urban agglomerations, the selection of tourism destination can be divided into two stages: first, select a city as the destination, and then select a scenic spot of the city as the tourism destination.

3.1.1. **The first stage**

The first stage of travel is conducted between cities at the scale of urban agglomerations, whether after the tourists outside the urban agglomerations enter the scope of urban agglomerations or when the tourists within the urban agglomerations start to travel. If each city in an urban agglomeration is regarded as a travel node and the transportation mode between cities is regarded as the edge of the connection node, the first stage of travel can be simplified as a sub-network among cities. At this time, the choice of travel starting point for tourists is essentially dependent on the city travel mode selected. Tourists are restricted by the city's external travel mode and cannot change the city travel starting point. Therefore, the city travel starting point can be regarded as the travel endpoint with fixed demand. The choice of travel destination in the first stage is a complex decision-making process for a city within an urban agglomeration. The comprehensive effect of the popularity, transportation convenience, economy, personal preference, and other factors of each city affects the selection decision-making process. Moreover, the attractiveness, transportation convenience, and other factors of each city change dynamically with the number of tourists. Therefore, the travel demand at the terminal is dynamic and variable.

3.1.2. **The second stage**

The ultimate destination of travel is scenic spots. After tourists choose a city as their destination, they will further select a scenic spot in the city as their final destination, which is the stage of travel after tourists enter a city. The transfer nodes (as the starting point to trip) in the city may be the city's entrance...
and exit (such as a high-speed toll station), the city's passenger transfer hub (tourism distribution center), etc., and the scenic spots are the travel destination. The travel demand at the starting point depends on the travel selection results in the first stage and can also be regarded as the travel endpoint with fixed demand; the choice of travel destination is also a complex decision-making behavior. The attraction degree of scenic spots, transportation convenience, and other factors will also change dynamically with the number of tourists. Therefore, the travel demand at the end of the second stage is also dynamic and variable. Tourists' choice of travel destination can not only rely on simple mathematical statistics methods, but also need to establish a mathematical model, calibrate the model parameters with a non-collective method, and reflect the dynamic and variable characteristics of travel demand with a collective approach.

3.1.3. Nested structure features of two-stage terminal selection

The attraction of scenic spots in different cities to tourists or the selection preference of tourists are different, and the correlation between various destinations should be fully considered. If the city where each scenic spot belongs is regarded as a nest, the two-layer nested structure (Ma et al., 2020; Paredes-Garcia et al., 2019) can be used to study the travel destination selection behavior at the scale of urban agglomeration. As shown in Figure 2, the upper level is the category of destination (nest), and the lower level is all optional destinations. Each destination belongs to a different nest.

3.2. Super network based on two-stage terminal selection

In order to simplify the two-stage selection problem of travel destination at the scale of urban agglomeration into the issue of the same transportation network, the concept of the super network was introduced (He et al., 2018), and the original transportation network was transformed into a separate network, as shown in Figure 3, which includes inter-city sub-network and intra-city sub-network. In the transportation network, the link in the traditional transportation network graph represents the transportation route of a certain means of transportation (or transportation mode). The node represents the transfer point or the starting and ending point of travel. The impedances of each transportation line include the travel time of the tourists in the vehicles, the waiting time of the transfer points, comfort and safety of the vehicles, and the ticket price.

Fig. 3. Super network based on two-stage terminal selection

Define the tourism transportation network \( G = (V, L) \), where \( V \) is the node-set, and \( L \) is the transportation line-set; \( l \) is a transportation line, \( l \in L \); \( O \) is the starting node-set, \( O \subset V \), and \( D \) is the ending node-set, \( D \subset V \); \( r \) is the starting node, and \( r \in O \); \( s \) is the ending node, \( s \in D \); assuming that the urban agglomeration is composed of \( M \) cities, \( M \subset V \), \( m \) is the termination node of the inter-city sub-network and \( m' \) is the starting node of the inter-city sub-network, \( m \in M, m' \in M \). \( m \rightarrow m' \) is a virtual transportation line with zero cost. \( W \) is the set of OD pairs in the transportation network. For the convenience of expression, \( a \) and \( b \) correspond to the inter-city sub-network and the inter-city sub-network, respectively.

Fig. 2. Nested structure diagram of destination selection
$W_a$ is used to represent the set of OD pairs in the inter-city sub-network, $W_b$ means the set of OD pairs in the inter-city sub-network, and $A$ and $B$ describe the set of transportation routes in the inter-city sub-network and the inter-city sub-network, respectively. $K_{rs}$ is a collection of all transportation routes between OD pairs $rs$, and $k_{rs} \in K_{rs}$ is a transportation route between OD pairs $rs$; $q_{rs}$ is the tourism transportation demand between OD pairs $rs$; $f_k$ refers to the tourism transportation volume on the transportation route $k \in K_{rs}$ between OD pairs $rs$; $x_i$ is the amount of tourist traffic on transport line $l$; $\delta_{lk}$ is the relationship between the transportation line and the transportation route. If the transportation line $l$ is on the route $k$, $\delta_{lk}$ is 1; otherwise, it is 0; $c_i(x_i)$ is the transportation cost on the transportation line $l$, it is assumed that it is a monotonic increasing function of the transportation volume $x_i$ on the transportation line $l$, and the functional form as shown in formula (1) is adopted.

$$c_i(x_i) = c_i^0 + \theta t_i^0 [1 + \tau (x_i / N_i)^\sigma]$$

(1)

where, $c_i^0$ is the direct cost of the transportation line $l$, mainly refers to the ticket price, toll, etc., which is generally fixed; $\theta$ is the time value of tourists (unit: yuan/min), it can be obtained by the method provided in the literature (Liu et al., 2021); $t_i^0$ is the travel time for tourists to use the transportation line $l$, including the travel time and waiting time in the vehicle, assuming that the impact of urban road traffic has been taken into account; $N_i$ is the capacity of the transportation mode on the transportation line $l$, i.e. the maximum number of passengers that can be transported per unit time; $\tau$, $\sigma$ are the parameters to be calibrated, generally take $\tau = 0.15$, $\sigma = 4$.

4. Two-stage equilibrium conditions

If the total travel generation amount of the known starting node $r$ is $O_r$, $\forall r$, $q_{rs}$ means the tourism transportation demand of the inter-city sub-network, and $q_{rs,m}$ means the transportation demand of the city $m$ in the transportation network, then it meets the constraint $O_r = \sum_{m \in M} q_{rm}$, $q_{rm} = \sum_{m \in D} q_{rs,m}$ and $\forall r \in O$. The travel demand matrix $Q = \{q\}$ reflects the variability of travel demand, i.e. considering how the tourists at the starting node $r$ choose the travel destination. $\phi_{rm}$ and $\phi_{rs,m}$ are used to represent respectively the attractiveness measurement of the inter-city sub-network terminal point and the sub-network terminal point in city $m$, i.e., the former represents the attractiveness measurement of city $m$ to tourists, and the latter represents the attractiveness measurement of the scenic spot $s$ in city $m$ to tourists. Assuming that all attractiveness measures can be converted into cost equivalent units of measurement, the higher the value of attractiveness measures, the stronger the attractiveness. Therefore, tourists always try to choose the travel destination with the minimum cost impedance and the maximum attraction measurement, i.e., $\mu_{rs} - \phi_{rm}$ and $\mu_{rs,m} - \phi_{rs,m}$ minimum destination, which is defined as the comprehensive tourism impedance.

Based on the analysis of travel choice behavior characteristics of urban agglomerations, the travel start demand of tourism passenger flow can be regarded as fixed, the travel end demand is flexible and variable, and the travel destination selection can be divided into two stages. Therefore, a travel demand function for tourists to choose the endpoint can be defined to reflect the change in travel demand. In order to reflect the attribute differences of the travel destination itself and tourists' preference for urban agglomeration, this paper introduces NL (Nested Logit) model (Yan et al., 2020; Kim et al., 2020) to describe tourists' choice of travel destination. It obtains the travel demand function taking the scenic spot $s$ of city $m$ as the destination as follows:

$$q_{rs,m} = O_r \cdot \frac{\exp\left[-\alpha (\mu_{rs} - \phi_{rm})\right]}{\sum_{m \in M} \exp\left[-\alpha (\mu_{rm} - \phi_{rm})\right]} \cdot \frac{\exp\left[-\beta (\mu_{rs,m} - \phi_{rs,m})\right]}{\sum_{s \in D} \exp\left[-\beta (\mu_{rs,m} - \phi_{rs,m})\right]}, \forall r \in O$$

(2)
In the formula, \( \alpha \) and \( \beta \) are the parameters to be calibrated in the model, \( \frac{\alpha}{\beta} \) is the layer scale parameters of the nested Logit model, meet \( 0 < \frac{\alpha}{\beta} \leq 1 \), and when \( \alpha=\beta \), it then degrades to the multinomial Logit model.

**Definition 1**, (two-stage travel destination selection and balanced assignment conditions) under the balanced state, in the passenger transport network of urban agglomeration, no tourists can reduce their travel costs by changing the travel route, and the choice of travel destination meets the nested Logit model (2).

Definition 1 includes two kinds of equilibrium, one is the certainty equilibrium of transportation route selection, and the other is the nested Logit random equilibrium of two-stage travel destination selection, which can be described by the following mathematical expressions:

\[
(c_k^a - \mu_{rm}) f_k = 0, \quad c_k^a - \mu_{rm} \geq 0, \quad \forall k \in K^a, \omega \in W, m \in M
\]

\[
(c_k^b - \mu_{ms}) f_k = 0, \quad c_k^b - \mu_{ms} \geq 0, \quad \forall k \in K^b, \omega \in W, m' \in M
\]

\[
q_{rs,m} = O_m - \sum_{m,M} \exp\left[-\alpha (\mu_{rm} - \varphi_{rm})\right] - \sum_{m,M} \exp\left[-\beta (\mu_{rm} - \varphi_{rm})\right], \quad \forall r \in O
\]

Among them, the cost \( c_k^a = \sum_{i \in A} c_i(x_i) \delta_{ik} \) of transportation route \( k \in K^a_{rm} \) between OD pairs \( rm \) in the inter-city sub-network, the cost \( c_k^b = \sum_{i \in B} c_i(x_i) \delta_{ik} \) of transportation route \( k \in K^b_{ms} \) between OD pairs \( ms \) in the sub-network of city m, and the cost \( c_k = \sum_{i \in A} c_i(x_i) \delta_{ik} + \sum_{i \in B} c_i(x_i) \delta_{ik} \) of transportation route \( k \in K_{rs,m} \) between OD pairs \( rs \) in the urban agglomeration network, \( \mu_{rm} \) is the minimum cost between OD pairs \( rm \) in the inter-city sub-network, \( \mu_{ms} \) is the minimum cost between OD pairs \( ms \) in the sub-network of city m and \( \mu_{rs,m} \) is the minimum cost of using city m between OD pairs \( rs \) in the urban agglomeration network, \( \mu_{rs,m} = \mu_{rm} + \mu_{ms} \).

5. **Two-stage balanced distribution model**

\[
\min Z(x, q) = \sum_{(r,m) \in W} \left[ c_i^a(\omega) + c_i^b(\omega) \right] d\omega
\]

\[
+ \frac{1}{\alpha} \left( q_{rm} \ln \frac{q_{rm}}{O_m} - O_m \right) - \sum_{(r,m) \in W} \varphi_{rm} q_{rm}
\]

\[
+ \frac{1}{\beta} \left( \sum_{m,M \in D} (q_{rs,m} \ln \frac{q_{rs,m}}{q_{rm}} - q_{rm}) \right) \sum_{m,M \in D} \varphi_{rs,m} q_{rs,m}
\]

s.t.

\( O_m = \sum_{m \in M} \lambda_r, \quad \forall r \in O, (\lambda_r) \)

\( q_{rm} = \sum_{m \in M} \lambda_{rs,m}, \quad \forall r \in O, (\lambda_{rs,m}) \)

\( \sum_{k \in K^a_{rm}} f_k = q_{rm}, \quad \forall (r,m) \in W, (\mu_{rm}) \)

\( \sum_{k \in K^b_{ms}} f_k = q_{rs,m}, \quad \forall (r,s) \in W, m \in M, (\mu_{rs,m}) \)

\( f_k \geq 0, \quad \forall k \in \left\{ K^a_{rs,m}, k \in K^b_{rs,m} \right\}, \quad (r,m) \in W_a, (r,s) \in W, r \in O \)

\( q_{rm} \geq 0, \quad \forall (r,m) \in W_a \)

\( q_{rs,m} \geq 0, \quad \forall (r,s) \in W \)

The illustrative constraints are as follows:

\( x_i = \sum_{(r,s) \in W} f_k \delta_{ik}, \forall l \in (A,B) \)

\( q_m = \sum_{r \in O} q_{rm}, \quad \forall m \in M \)

\( q_s = \sum_{r \in W, \forall s \in D} q_{rs,m} \)
Proof: K-T condition of the model (6), get
\[
\left\{ \sum_{i \in A, B} \left[ c_i^b(x_i) + c_i^s(x_i) \right] \delta_{i,k} - \mu_{rs,m} \right\} f_k = 0,
\]
\[
\sum_{i \in A, B} \left[ c_i^b(x_i) + c_i^s(x_i) \right] \delta_{i,k} - \mu_{rs,m} \geq 0,
\]
\[
k \in K_{rs,m}, (r,s) \in W
\]  
(17)

Because \( \mu_{rs,m} = \mu_{rm} + \mu_{ms} \), formula (17) can be changed into formula (20) and formula (21)
\[
\left\{ \sum_{i \in A} c_i^a(x_i) \delta_{i,k} - \mu_{rm}^a \right\} f_k = 0,
\]
\[
\sum_{i \in A} c_i^a(x_i) \delta_{i,k} - \mu_{rm}^a \geq 0,
\]
\[
k \in K_{rm}^a, (r,m) \in W_a
\]  
(20)

\[
\left\{ \sum_{i \in B} c_i^b(x_i) \delta_{i,k} - \mu_{ms}^b \right\} f_k = 0,
\]
\[
\sum_{i \in B} c_i^b(x_i) \delta_{i,k} - \mu_{ms}^b \geq 0,
\]
\[
k \in K_{ms}^b, (m,s) \in W_b
\]  
(21)

Formulas (20) - (21) indicate that in the tourism transportation network of the urban agglomeration, the selection of transportation route by tourists meets the user equilibrium conditions (3) - (4).

From formula (18), we can get
\[
\text{If } q_{rm} > 0 \text{ then } \left( \frac{1}{\alpha} \ln \frac{q_{rm}}{O_r} - \varphi_{rm} \right) - \lambda_r + \mu_{rm} = 0,
\]
\[
\forall r \in O,(r,m) \in W_a
\]  
(22)

get
\[
q_{rm} = \exp(\alpha \lambda_r) \cdot \exp[-\alpha(\mu_{rm} - \varphi_{rm})],
\]
\[
\forall r \in O,(r,m) \in W_a
\]  
(23)

Summation to \( m \), focus on conservation conditions (9), get
\[
1 = \exp(\alpha \lambda_r) \cdot \sum_{m \in M} \exp[-\alpha(\mu_{rm} - \varphi_{rm})],
\]
\[
\forall r \in O,(r,m) \in W_a
\]  
(24)

Therefore, the \( \lambda_r \) is actually the expected travel cost for tourists to choose city \( m \) for travel. By substituting \( \exp(\alpha \lambda_r) \) of the formula (24) in formula (23), the Logit model of selecting city \( m \) as the destination in the inter-city sub-network is obtained:
\[
q_{rm}^a = \frac{\exp[-\alpha(\mu_{rm} - \varphi_{rm}^a)]}{\sum_{m \in M} \exp[-\alpha(\mu_{rm} - \varphi_{rm}^a)]},
\]
\[
\forall r \in O,(r,m) \in W_a
\]  
(25)

Similarly, from formula (19), get
\[
1 = \exp(\beta \lambda_{rm}) \cdot \sum_{s \in D} \exp[-\beta(\mu_{rs,m} - \varphi_{rs,m})],
\]
\[
\forall (r,s) \in W, m \in M
\]  
(26)

Therefore, the \( \lambda_{rm} \) is actually the expected travel cost for tourists to choose the destination \( s \) within city \( m \). By substituting \( \exp(\beta \lambda_{rm}) \) of the formula (26) in formula (25), the Logit model of selecting the travel destination \( s \) in the sub-network of city \( m \) is obtained:
\[
q_{rs,m} = \frac{\exp[-\beta(\mu_{rs,m} - \varphi_{rs,m})]}{\sum_{s \in D} \exp[-\beta(\mu_{rs,m} - \varphi_{rs,m})]},
\]
\[
\forall (r,s) \in W, m \in M
\]  
(27)

By multiplying formula (25) and formula (27), the nested Logit model of tourists choosing a scenic spot of city \( m \) in the urban agglomeration network is obtained as follows:
\[ g_{rm}, q_{rs,m} = q_{rs,m} = \]
\[ \frac{\exp[-\alpha(\mu_{rs,m} - \phi_{rs,m})]}{1 + \sum_{m \in M} \exp[-\alpha(\mu_{rs,m} - \phi_{rs,m})]} \]
\[ \frac{\exp[-\beta(\mu_{rs,m} - \phi_{rs,m})]}{1 + \sum_{s \in D} \exp[-\beta(\mu_{rs,m} - \phi_{rs,m})]} \]  

That is, two-stage equilibrium condition (5), the approval is completed.

6. Solution algorithm

The feasible region of the mathematical model (6) is composed of linear constraints, so the feasible domain is a compact convex set, and the function \( c_i(x_i) \), \( \ln g_{rm} \), \( \ln q_{rs,m} \) is continuous. According to the Brouwer fixed point theory, it is known that the model (VI) has at least one solution, and then the monotony assumption of \( c_i(x_i) \) indicates that the model (6) has a unique solution. Based on MSA (Method of Successive Average) (Botte et al., 2020), the following solution algorithm is designed:

Step 1: initialization, calculate various expenses between OD pairs according to the "zero flow" transportation line time, complete the division of \( q_{rm}^{(1)} \) and \( q_{rs,m}^{(1)} \) of initial and terminal demand with the nested Logit model, and then allocate the transportation to obtain the initial transportation line flow \( x_i^{(1)} \) and \( l \in \{A, B\} \), set the iteration number \( n=1 \).

Step 2: calculate the transportation cost \( c_i^{(n)} = c_i^{(a)}(x_i^{(n)}) + c_i^{(b)}(x_i^{(n)}) \) and \( l \in \{A, B\} \).

Step 3: On the basis of \( \{c_i^{(n)}\} \) and \( \{c_i^{(a(n))}\} \), in the inter-city sub-network, find the shortest transportation route from each point to all the destinations, and calculate the cost \( \mu_{rm}^{(n)} \). In the network of urban agglomerations, the shortest transportation route from each starting point through city \( m \) to all destinations is found and its cost \( \mu_{rs,m}^{(n)} \) is calculated.

Step 4: calculate the auxiliary travel demand \( g_{rm}^{(n)} \) and \( g_{rs,m}^{(n)} \).

\[ g_{rm}^{(n)} = O_r \cdot \frac{\exp[-\alpha(\mu_{rm}^{a} - \phi_{rm}^{a})]}{\sum_{m \in M} \exp[-\alpha(\mu_{rm}^{a} - \phi_{rm}^{a})]} \]
\[ g_{rs,m}^{(n)} = O_r \cdot \frac{\exp[-\beta(\mu_{rs,m}^{a} - \phi_{rs,m}^{a})]}{\sum_{s \in D} \exp[-\beta(\mu_{rs,m}^{a} - \phi_{rs,m}^{a})]} \]

Step 5: \( g_{rm}^{(n)} \) and \( g_{rs,m}^{(n)} \) are used to make deterministic user equilibrium assignment in the inter-city sub-network and the intra-city sub-network to obtain the passenger flow \( y_{rl}^{(n)} \) and \( l \in \{A, B\} \) of the auxiliary transportation lines in the equilibrium state.

Step 6: update various traffic with the MSA method
\[ q_{rm}^{(n+1)} = q_{rm}^{(n)} + \frac{1}{n+1} (g_{rm}^{(n)} - q_{rm}^{(n)}) \]
\[ q_{rs,m}^{(n+1)} = q_{rs,m}^{(n)} + \frac{1}{n+1} (g_{rs,m}^{(n)} - q_{rs,m}^{(n)}) \]
\[ x_{i}^{(n+1)} = x_{i}^{(n)} + \frac{1}{n+1} (y_{i}^{(n)} - x_{i}^{(n)}) \]

Step 7: If meeting the convergence requirements, terminate the algorithm; otherwise, let \( n=n+1 \), and return to step 2.

7. Example analysis

A tourism transportation network of urban agglomerations composed of 3 cities and 6 scenic spots was designed to illustrate the feasibility of the model. As shown in Fig. 4, the transportation network has 9 nodes and 24 transportation lines (sections). Nodes A, B and C represent each city node of the urban agglomeration, and nodes A1, A2, B1, B2, C1 and C2 represent each scenic spot node in the city; transportation lines 1-6 is a two-way arc of the inter-city sub-network, and 7-24 is a one-way arc of the intra-city sub-network.

The basic data of the defined transportation network is shown in Table 1, and the attraction measurement data between the starting and ending points of the travel is shown in Table 2. The total demand for the starting point of travel in each city is \( O_A = 3000 \) person / hour, \( O_B = 4000 \) person / hour, and \( O_C = 5000 \)
person / hour, respectively (assuming that the travel volume in the total demand for the starting point of travel in each city has been separated). Other parameters in the model are \( \alpha=0.01, \beta=0.1 \) and \( \theta=2 \). The cost impedance of the transportation lines is in the form of the formula (1). The termination index of the algorithm is set to 0.001. The mathematical planning model (6) is used to calculate the test network. After 14 iterations, the accuracy requirements are met. The travel demand of each OD point and the distribution results of the tourist flow of each transportation line are shown in Table 3 and table 4.

![Simplified diagram of tourism transportation network in urban agglomeration](image)

**Fig. 4.** Simplified diagram of tourism transportation network in urban agglomeration

<table>
<thead>
<tr>
<th>Transportation route No.</th>
<th>Ticket price ( c_i^0 ) (yuan)</th>
<th>Travel time ( t_i^0 ) (min)</th>
<th>Transport capacity ( N_i ) (person / hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>210</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>140</td>
<td>1000</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>90</td>
<td>1500</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>270</td>
<td>800</td>
</tr>
<tr>
<td>5</td>
<td>140</td>
<td>100</td>
<td>1500</td>
</tr>
<tr>
<td>6</td>
<td>90</td>
<td>210</td>
<td>500</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>50</td>
<td>1000</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>25</td>
<td>1500</td>
</tr>
<tr>
<td>9</td>
<td>25</td>
<td>35</td>
<td>1000</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>30</td>
<td>1000</td>
</tr>
<tr>
<td>11</td>
<td>20</td>
<td>15</td>
<td>1500</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td>25</td>
<td>1000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transportation route No.</th>
<th>Ticket price ( c_i^0 ) (yuan)</th>
<th>Travel time ( t_i^0 ) (min)</th>
<th>Transport capacity ( N_i ) (person / hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>8</td>
<td>60</td>
<td>1500</td>
</tr>
<tr>
<td>14</td>
<td>80</td>
<td>40</td>
<td>1500</td>
</tr>
<tr>
<td>15</td>
<td>45</td>
<td>50</td>
<td>1000</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>35</td>
<td>1500</td>
</tr>
<tr>
<td>17</td>
<td>45</td>
<td>20</td>
<td>1500</td>
</tr>
<tr>
<td>18</td>
<td>25</td>
<td>30</td>
<td>1000</td>
</tr>
<tr>
<td>19</td>
<td>6</td>
<td>50</td>
<td>1500</td>
</tr>
<tr>
<td>20</td>
<td>75</td>
<td>40</td>
<td>1500</td>
</tr>
<tr>
<td>21</td>
<td>40</td>
<td>45</td>
<td>1000</td>
</tr>
<tr>
<td>22</td>
<td>2</td>
<td>20</td>
<td>1500</td>
</tr>
<tr>
<td>23</td>
<td>30</td>
<td>15</td>
<td>1500</td>
</tr>
<tr>
<td>24</td>
<td>15</td>
<td>30</td>
<td>1000</td>
</tr>
</tbody>
</table>

**Table 1.** Basic data table of tourism transportation network in urban agglomeration
Table 2. Data table of attraction measurement between travel start and endpoints

<table>
<thead>
<tr>
<th>Start-end points $r \rightarrow m$</th>
<th>Start-end points through city $r \rightarrow s, m$</th>
<th>Start-end points $r \rightarrow m$</th>
<th>Start-end points through city $r \rightarrow s, m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A → A</td>
<td>A → A, A</td>
<td>30 A → A</td>
<td>60 C → A</td>
</tr>
<tr>
<td></td>
<td>A → A, A</td>
<td></td>
<td>C → A</td>
</tr>
<tr>
<td>A → B</td>
<td>A → B, B</td>
<td>80 A → B</td>
<td>110 C → B</td>
</tr>
<tr>
<td></td>
<td>A → B, B</td>
<td></td>
<td>C → B</td>
</tr>
<tr>
<td>A → C</td>
<td>A → C, C</td>
<td>130 A → C</td>
<td>50 C → C</td>
</tr>
<tr>
<td></td>
<td>A → C, C</td>
<td></td>
<td>C → C</td>
</tr>
<tr>
<td>B → A</td>
<td>B → A, A</td>
<td>70 B → A</td>
<td>40 B → B</td>
</tr>
<tr>
<td></td>
<td>B → A, A</td>
<td></td>
<td>B → B</td>
</tr>
<tr>
<td>B → C</td>
<td>B → C, C</td>
<td>120 B → C</td>
<td>110 B → C</td>
</tr>
</tbody>
</table>

Table 3. Results of trip distribution in the experimental network (unit: person / hour)

<table>
<thead>
<tr>
<th>Start-end points $r \rightarrow m$</th>
<th>Start-end points through city $r \rightarrow s, m$</th>
<th>Start-end points $r \rightarrow m$</th>
<th>Start-end points through city $r \rightarrow s, m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A → A</td>
<td>A → A, A</td>
<td>1247 A → A</td>
<td>1270 C → A</td>
</tr>
<tr>
<td></td>
<td>A → A, A</td>
<td></td>
<td>C → A</td>
</tr>
<tr>
<td>A → B</td>
<td>A → B, B</td>
<td>684 A → B</td>
<td>1798 C → B</td>
</tr>
<tr>
<td></td>
<td>A → B, B</td>
<td></td>
<td>C → B</td>
</tr>
<tr>
<td>A → C</td>
<td>A → C, C</td>
<td>1069 A → C</td>
<td>1932 C → C</td>
</tr>
<tr>
<td></td>
<td>A → C, C</td>
<td></td>
<td>C → C</td>
</tr>
<tr>
<td>B → A</td>
<td>B → A, A</td>
<td>1006 B → A</td>
<td>1573 B → B</td>
</tr>
<tr>
<td></td>
<td>B → A, A</td>
<td></td>
<td>B → B</td>
</tr>
<tr>
<td>B → C</td>
<td>B → C, C</td>
<td>1421 B → C</td>
<td>1421 B → C</td>
</tr>
</tbody>
</table>

Table 4. Results of tourist flow assignment in the experimental network (unit: person / hour)

<table>
<thead>
<tr>
<th>Transportation line number</th>
<th>Passenger flow</th>
<th>Transportation line number</th>
<th>Passenger flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (A→B)</td>
<td>318</td>
<td>10</td>
<td>698</td>
</tr>
<tr>
<td>2 (A→B)</td>
<td>366</td>
<td>11</td>
<td>791</td>
</tr>
<tr>
<td>3 (B→C)</td>
<td>934</td>
<td>12</td>
<td>702</td>
</tr>
<tr>
<td>4 (B→C)</td>
<td>487</td>
<td>13</td>
<td>905</td>
</tr>
<tr>
<td>5 (C→A)</td>
<td>806</td>
<td>14</td>
<td>698</td>
</tr>
<tr>
<td>6 (C→A)</td>
<td>464</td>
<td>15</td>
<td>812</td>
</tr>
<tr>
<td>1 (B→A)</td>
<td>422</td>
<td>16</td>
<td>651</td>
</tr>
<tr>
<td>2 (B→A)</td>
<td>584</td>
<td>17</td>
<td>497</td>
</tr>
<tr>
<td>3 (C→B)</td>
<td>1175</td>
<td>18</td>
<td>492</td>
</tr>
<tr>
<td>4 (C→B)</td>
<td>623</td>
<td>19</td>
<td>489</td>
</tr>
<tr>
<td>5 (A→C)</td>
<td>678</td>
<td>20</td>
<td>431</td>
</tr>
<tr>
<td>6 (A→C)</td>
<td>391</td>
<td>21</td>
<td>447</td>
</tr>
<tr>
<td>7</td>
<td>404</td>
<td>22</td>
<td>1270</td>
</tr>
<tr>
<td>8</td>
<td>470</td>
<td>23</td>
<td>1022</td>
</tr>
<tr>
<td>9</td>
<td>458</td>
<td>24</td>
<td>763</td>
</tr>
</tbody>
</table>
Fig. 5. Each tourist demand under different $\alpha$.

Fig. 6. Each tourist demand under different $\beta$. 
The inter-layer proportional parameters $\frac{\alpha}{\beta}$ of the travel demand distribution model of nested Logit reflect the correlation degree of the transportation route selection branches under each destination category. The closer $\alpha$ is to $\beta$, the smaller the correlation between the transportation route selection branches under the end of the same category is; the closer $\alpha$ is to 0, the greater the correlation among the selected branches. Taking travel starting point A as an example, Fig. 5 shows the calculation results of travel demand distribution when $\beta$ is fixed at 0.1 and $\alpha$ changes, and Fig. 6 shows the calculation results of travel demand distribution when $\alpha$ is fixed at 0.01 and $\beta$ changes. The results show that: ① with the increase of $\alpha$ or $\beta$, the overall trend is that the demand for scenic spots of city A increases significantly, while the demand for scenic spots of cities B or C decreases. Which is because the larger $\alpha$ or $\beta$, the closer the tourists' choice of the destination to the certainty choice, they will wisely choose the transportation route with the least comprehensive tourism impedance, the influence of the city C attraction measurement will gradually weaken, and the tourists will choose the transportation route with the less comprehensive tourism impedance more clearly, which makes the demand of the tourist attractions $C_1$ and $C_2$ decrease most significantly, while the tourist comprehensive impedance of the scenic spot of city A is the least, which makes the scenic spots $A_1$ and $A_2$ increase most significantly;② when $\alpha=\beta$, the nested Logit was reduced to the multinomial Logit model. Compared with the calculation results of $\alpha=0.01$ and $\alpha=0.1$, it can see that in the multinomial Logit model, tourists only select the transportation route with the maximum comprehensive utility. The nested Logit model fully considers the attractiveness of each city, reflecting the correlation between the transportation route selection branches under the same category, which is more in line with the behavior of tourists when choosing the transportation route; ③ compared with the calculation results of changes $\alpha$ and $\beta$, it is found that Fig. 5 is more obvious than Fig. 4 in terms of changes in data, indicating that the impact of $\alpha$ is greater than $\beta$ on travel demand. Compared with the traditional model of $\alpha=\beta$, the model in this paper is closer to the real travel choice state.

8. Conclusion
This paper studies the transportation problem of tourism passenger flow at the scale of urban agglomerations, establishes the balanced distribution model of tourism passenger flow transportation combination with fixed starting demand and variable terminal demand in two stages, designs the algorithm to solve the model, and illustrates the effectiveness of the model and the calculation method through examples. The example results show that compared with the multinomial Logit model, the combined model of travel distribution and transportation assignment based on the nested Logit model can better reflect the two-stage selection behavior of travel destination of tourists under the scale of urban agglomerations.

The inter-layer ratio parameters $\frac{\alpha}{\beta}$ of the nested Logit model can reflect the correlation degree of the selected branches of transportation routes under each destination category, and $\alpha$ has a more significant impact than $\beta$ on the travel demand. The results of the theoretical analysis in this paper are very meaningful for the analysis of the travel destination selection behavior of tourists at the scale of urban agglomerations. They are expected to provide some ideas for the allocation of tourism resources, tourism transportation services and other issues within the scope of urban agglomerations.

Acknowledgement
The work is supported by Funded by Intelligent Policing Key Laboratory of Sichuan Province (No. ZNJW2022ZZMS003, No. ZNJW2022KFQN004).

Reference


[18] Liu, Peng., Liu, J., Ghim, P. O. & Tian, Q.(2020). Flow pattern and optimal capacity in a bi-modal traffic corridor with heterogeneous users. Transportation Research Part E, 2020,


