CONFLICT-FREE TRAJECTORY PLANNING BASED ON THE MODEL PREDICTIVE CONTROL THEORY

Han Yun-xiang[1], Huang Xiao-qiong[2]

[1]Jiangsu University of Technology, School of automobile and traffic engineering, Changzhou, P.R China

e-mail: hannuaa@126.com

[2]Jiangsu University of Technology, School of business, Changzhou, P.R China

e-mail: huangxiaoqiong78@126.com

Abstract: *Model Predictive Control (MPC) is a model-based control method based on a receding horizon approach and online optimization. A key advantage of MPC is that it can accommodate constraints on the inputs and outputs. This paper proposes a max-plus general modeling framework adapted to the robust optimal control of air traffic flow in the airspace. It is shown that the problem can be posed as the control of queues with safety separation-dependent service rate. We extend MPC to a class of discrete-event system that can be described by models that are linear in the max-plus algebra with noise or modeling errors. Regarding the single aircraft as a batch, the relationships between input variables, state variables and output variable are established. We discuss some key properties of the system model and indicate how these properties can be used to analyze the behavior of air traffic flow. The model predictive control design problems are defined for this type of discrete event system to help prepare the airspace for various robust regulation needs and we give some extensions of the air traffic max-plus linear systems.*

Key words: *civil aviation, air transportation, aircraft, air traffic control, separation, trajectories, optimization, model predictive control*

1. Introduction

Air traffic flow is characterized by ever tighter time specifications, increasing airspace capacity and decreasing air traffic controller workloads. Although current air traffic systems are already highly sophisticated, the aircraft conflict detection and resolution has been the main research field in the last decade, following the sustained growth of air traffic. The increase of air traffic volume urges to improve the efficiency of Air Traffic Control Management as the sky becomes saturated, which was attested by the Next Generation Air Transportation System (NGATS) and the Single European Sky Air Traffic Research System (SESAR) aimed at highlighting the importance of strategic de-confliction (Swenson et al., 2006; Dlugi et al., 2007; Schuster & Ochieng, 2014).

The aircraft conflict resolution maneuvers are based on conflict detection results which involve estimating an aircraft's future trajectory by using the related flight parameters. The term strategic de-confliction is often used to define actions taken when the aircraft takeoff time is known with

sufficient accuracy or even after the flight is airborne but with sufficient time to allow a collaborative decision-making process to occur (Ruiz et al., 2014). This term excludes tactical instructions and clearances that require an immediate response but including activities such as dynamic route allocation. Typically, methods to solve the problems of conflict detection can be categorized into three kinds on the whole: nominal, worst case and probabilistic (Kuchar & Yang, 2000). Furthermore, the aircraft conflict resolution algorithms can be categorized into pair-wise strategy and global strategy. Before our work, former studies aimed at presenting a Constraint Programming model of this large scale combinatorial optimization problem, providing that aircraft are able to follow their trajectories accurately (Barnier & Allignol, 2011). For the collaborative planning process including more enriched information of multi-aircraft de-confliction trajectories, the strategic de-confliction algorithm based on causal modeling was also put forward (Ruiz et al., 2014). In addition, research on the tactical conflict resolution problem has

traditionally focused on developing open-loop optimal policies for regulating aircraft operations. A number of conflict avoidance models have been examined, notably the mixed-integer linear programming algorithm and genetic algorithms (Omer, 2015; Durand et al., 1996)). Besides, some optimal control approaches were also used to solve this problem (Pallottino et al., 2002; Hu et al., 2002; Raghunathan et al., 2003; Clements, 1999; Friedman, 1988, Friedman 1991; Vela et al., 2010; Matsuno et al., 2015; Alliot et al., 2000). However, unlike some conflict-resolution models mentioned above, hybrid system model was also formulated to synthesize provably safe conflict resolution maneuvers (Tomlin et al., 1998, 2000, 2001). All these tasks are currently conducted manually by air traffic controllers, and contribute significantly to their workload. In order to meet the increasing traffic demand and reduce the workload of controllers at the tactical level, there is a desire to introduce a greater level of automation and decision support for air traffic management at the strategic level.

Air traffic control system is a typical example of discrete event system essentially, which changes due to the occurrence of events compared with continuous variable systems, whose behavior is governed by the progression of time or the ticks of a clock. Due to the logic characteristics of air traffic flow in the airspace network, the models that describe its behavior are nonlinear in conventional algebra models. However, there is a class of discrete event system that can be described by a model that is linear in the max-plus algebra framework in which only synchronization and no concurrency or choice occurs. The internal linear properties of max-plus models that describe air traffic flow make control policies for the airspace very attractive. Attempts like this have been made to manufacturing systems, telecommunication networks, railway networks and parallel computing (Goverde et al., 1999; Olsder, 1989; Olsder, 1993). In this paper, we will develop a max-plus-linear framework for air traffic system and the model can be used to control all resources of the airspace. The structure of this paper is organized as follows. We first give a concise introduction to max-plus basic theory and formulate the constraints of single jet route. Next, we present the constraints of multiple jet routes. The whole airspace max-plus coupled models are given in section 4. The multi-

aircraft trajectory optimization model and a typical worked example are provided in section 5 and section 6. We end with planned further works to enhance the approach.

2. Formulation of the single jet route traffic flow max-plus model

In this section, we firstly give the basic definition of the max-plus algebra and present some results on a class of $(\max,+)$ functions. The basic operations of the max-plus algebra are maximization and addition, which will be represented by $\oplus$ and $\otimes$, respectively (Cohen, 1999). Define $\varepsilon=-\infty$ and $R_\varepsilon=R\cup\{\varepsilon\}$, the basic operations addition $(\oplus)$ and multiplication $(\otimes)$ are defined as follows:

$$x\oplus y=\max(x,y) \tag{1}$$

$$x\otimes y=x+y \tag{2}$$

for numbers x, $y\in R_\varepsilon$ and

$$[A\oplus B]_{ij}=a_{ij}\oplus b_{ij}=\max(a_{ij},b_{ij}) \tag{3}$$

$$[A\otimes C]_{ij}=\bigoplus_{k=1}^{n}a_{ik}\otimes c_{kj}=\max_{k=1,2,\ldots,n}(a_{ik}+c_{kj}) \tag{4}$$

for matrices $A,B\in R_\varepsilon^{m\times n}$ and $C\in R_\varepsilon^{n\times p}$. The matrix $\mathcal{E}$ is the max-plus algebra zero matrix: $[\mathcal{E}]_{ij}=\varepsilon$ for all i, j.

To acquire the max-plus algebra model that we use for air traffic system, we start by deciding the points or lines through which traffic flow rates need to be determined. These boundaries, henceforth called control boundaries, can consist of split point, sector boundary point, airspace fixes, intersections of major jet routes, or other metering points. Adding more control point provides more decision support, but decreases the flexibility to adapt to factors not precisely modeled. A control unit is delimited by an input and output control boundary, that is, all the traffic flow associated with it enters and exits through the same control boundary. Consider, for example, a jet route denoted by AB illustrated in Fig.1 consisting of n sub-segments. We assume

that each sub-segment starts working as soon as possible, i.e., as soon as the aircraft waiting for service is available, and as soon as the sub-segment is idle. The nominal operation of the jet route follows a predesigned order and we assume that all the aircrafts follow a prescheduled route. Let n be the number of sub-segment $M_1, M_2, \cdots, M_n$ and m be the number of aircraft $P_1, P_2, \cdots, P_m$. Each sub-segment of the jet route has m aircrafts allocated to it and the length of each segment is minimum horizontal safety separation $d_{\min}$. In other words, each sub-segment $M_j(j = 1, 2, \ldots, n)$ provides sequential service for each aircraft $P_i(i = 1, 2, \ldots, m)$ and each aircraft $P_i(i = 1, 2, \ldots, m)$ accepts service provided by each sub-segment $M_j(j = 1, 2, \ldots, n)$. The service process provided by each sub-segment is called service activity. Aircraft and sub-segment are two types of shared resources in it. The circumstances that aircraft begins to enter into a certain sub-segment or sub-segment begins to provide service are all called resource input. In contrast, the aircraft flows out of a particular sub-segment or sub-segment completes the service are all called resource output. The number of independent service activities in the service process is mn whereas the number of resource input and resource output are both $(m+n)$. In the serial service process, the key issue is to deduce logic relation between input variable and output variable, from which we can obtain system state equation and output equation. The meaning of each variable is defined as follows (Van den Boom & de Schutter, 2002):

Definition x_{ij}: the earliest time instant at which the sub-segment $M_j(j = 1, 2, \ldots, n)$ starts to provide service for the aircraft $P_i(i = 1, 2, \ldots, m)$ in a specific batch;

Definition u_l: time instant at which the lth$(l = 1, 2, \ldots, n+m)$ resource involved in the first service activity in a specific batch;

Definition y_l: the earliest time instant at which the lth$(l = 1, 2, \ldots, n+m)$ resource releases from a specific batch;

Definition t_{ij}: time period during which the sub-segment $M_j(j = 1, 2, \ldots, n)$ provides service for the aircraft $P_i(i = 1, 2, \ldots, m)$ in a specific batch;

Besides, we should give some fundamental preconditions relevant to system service process before establishing specified system model. One common feature of the jet route is that each sub-segment cannot begin a fresh service activity until certain preceding sub-segments have all completed their service activities. For the serial service process mentioned above, M_j can provide service activity for P_i if and only if two premises are satisfied simultaneously: $M_j(j = 1, 2, \ldots, n)$ is available and $P_i(i = 1, 2, \ldots, m)$ is located at the entry of $M_j(j = 1, 2, \ldots, n)$. Furthermore, we can acquire the "time-logic" relations that the system should follow:

(1)For $i = 1$ and $j = 1$, the service prerequisite corresponds to sub-segment j is available and aircraft i is located at the entry of the sub-segment j, so it can be characterized as:
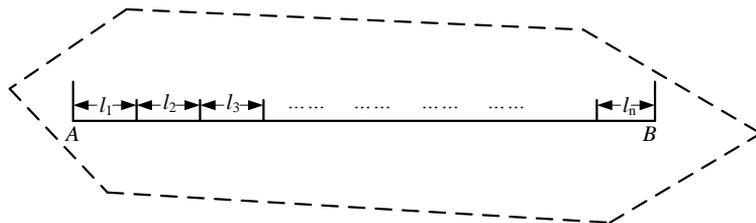
$$x_{11} = \max\{u_1, u_{n+1}\} \tag{5}$$

Fig. 1. The division of single jet route

(2) For $i=1$ and $j\neq1$, the service prerequisite corresponds to sub-segment j is available and sub-segment $(j-1)$ has completed the service activity for aircraft i, so it can be characterized as:

$$x_{1j} = \max\left\{u_j, x_{1,j-1} + t_{1,j-1}\right\} \qquad (6)$$

(3) For $i\neq1$ and $j=1$, the service prerequisite corresponds to sub-segment $(j+1)$ has completed the service activity for aircraft $(i-1)$ and aircraft i is located at the entry of the sub-segment j, so it can be characterized as:

$$x_{i1} = \max\left\{x_{i-1,2} + t_{i-1,2}, u_{n+i}\right\} \qquad (7)$$

(4) For $i\neq1$ and $j\neq1$, the service prerequisite corresponds to sub-segment $(j+1)$ has completed the service activity for aircraft $(i-1)$ and sub-segment $(j-1)$ has completed the service activity for aircraft i, so it can be characterized as:

$$x_{ij} = \max\left\{x_{i-1,j+1} + t_{i-1,j+1}, x_{i,j-1} + t_{i,j-1}\right\}$$
$$(i = 2,3,\cdots,m; j = 2,3,\cdots,n) \qquad (8)$$

According to what has been discussed above, the system model can be summarized as:

$$\begin{cases} \mathbf{x} = \mathbf{A}\otimes x \otimes \mathbf{B}\otimes\mathbf{u} \\ \mathbf{y} = \mathbf{C}\otimes x \end{cases} \qquad (9)$$

for:

$$\mathbf{u} = \left[u_1, u_2, \ldots, u_n, u_{n+1}, u_{n+2}, \ldots, u_{n+m}\right]^{\mathrm{T}}$$
$$\mathbf{y} = \left[y_1, y_2, \ldots, y_n, y_{n+1}, y_{n+2}, \ldots, y_{n+m}\right]^{\mathrm{T}}$$
$$\mathbf{x} = \left[x_{11}, x_{12}, \ldots, x_{1n}, x_{21}, x_{22}, \ldots, x_{2n}, \cdots, x_{m1}, x_{m2}, \ldots, x_{mn}\right]^{\mathrm{T}}$$

With the established air traffic flow state space model presented above, we can come to some conclusions listed below:

Remark 1 The proposed system model reveals the inherent linear cause-and-effect relationship between input variables and output variables in the service process. In addition, there exists a similarity between the conventional discrete time linear model and the max-plus linear model. The original system model can be transformed into:

$$\begin{cases} \bar{\mathbf{x}}(k) = \bar{\mathbf{A}}\otimes\bar{\mathbf{x}}(k-1)\oplus\bar{\mathbf{B}}\otimes\bar{\mathbf{u}}(k) \\ \bar{\mathbf{y}} = \bar{\mathbf{C}}\otimes\bar{\mathbf{x}}(k) \end{cases} \qquad (10)$$

by regarding the single aircraft as a batch and incorporating a buffer time parameter t_k, where k denotes batch number. Furthermore, we can acquire feedback control equation $\mathbf{u}(k) = \mathbf{K}\mathbf{y}(k-1)$ in which $\mathbf{K}$ is feedback constant;

Remark 2 The parameters in the proposed model have the characteristic of relative immobility. Given spatial configuration of jet routes, it doesn't change relative form of the max-plus system model when the entry sequence of aircraft changes and it just changes relevant matrix parameters. Intuitively, the order change of aircraft corresponds to the system state variables' displacement transformation;

Remark 3 The parameters t_{ij} and x_{ij} should be random variables when considering the influence of all kinds of random factors relevant to aircraft flight. In this way, we can transform the deterministic system model into stochastic ones;

Remark 4 The analytical form of state vector $\mathbf{x}$ and output vector $\mathbf{y}$ in the proposed max-plus system model can be expressed as follows:

$$\begin{cases} \mathbf{x} = \mathbf{A}^*\otimes\mathbf{B}\otimes\mathbf{u} \\ \mathbf{y} = \mathbf{C}\otimes\mathbf{A}^*\otimes\mathbf{B}\otimes\mathbf{u} \end{cases} \qquad (11)$$

where:

$$\mathbf{A}^* = \mathbf{E}\oplus\mathbf{A}\oplus(\mathbf{A}\otimes\mathbf{A})\oplus(\mathbf{A}\otimes\mathbf{A}\otimes\mathbf{A})\oplus\cdots\oplus$$
$$\oplus\left(\underbrace{\mathbf{A}\otimes\mathbf{A}\otimes\mathbf{A}\otimes\cdots\otimes\mathbf{A}}_{mn-1}\right)$$

3. Formulation of the multiple jet routes traffic flow max-plus model

Multiple jet routes with intersections air traffic flow max-plus model can be handled similarly compared with single jet route and the competition between different traffic flows passing through the intersections can be modeled by additional constraints. In the following section, we will focus on the formulation of cross jet routes traffic flow max-plus model following the dynamics of single jet route traffic flow. Due to limited space, here we will only discuss the most typical scenario depicted in Fig.2 and $|C_2D_1|=|C_1D_2|=|E_1F_2|=|E_2F_1|=d_{min}$. In addition, let n_1 and n_2 denote the number of divided sub-segments associated with A_1OB_1 and A_2OB_2 respectively, then the length of each sub-segment is d_{min} except sub-segment C_1F_1 and C_2F_2.

It is easy to know that the flight conflict point lies in each split point near the intersection O for the cross jet routes. If the sequential aircrafts have the same flight jet route, then it is similar to the case of single jet route. We mainly discuss the sequential aircrafts with different flight jet routes. For example, one aircraft called g is located on A_1OB_1 and the other called h is located on A_2OB_2. If aircraft g arrives O firstly, then we should impose the following constraint at the merge point O according to the rule of first come first service:

$$x_{l+2}(h)=\max\begin{pmatrix} x_{l+4}(h)+t_{l+4}(h),x_{l+2}(g)+ \\ +t_{l+2}(g)+t_h \end{pmatrix} \quad (12)$$

4. Formulation of airspace traffic flow max-plus model

In fact, a basic airspace unit (such as a terminal control area) may involve a wide variety of flight conflicts. It is straightforward to construct max-plus model adapted to the control of air traffic flows in the whole airspace using the basic max-plus models presented in the previous sections. From what has been discussed above, we can separate the whole airspace into distinct sections and the synthesis model can be regarded as the series connection result of single jet route traffic flow model. Consider, for example, an airspace incorporating two sub-sections, we can index each sub-model by their associated input and output boundaries as shown in Fig.3.

Thus, the synthesized system model can be described as:

$$\begin{cases} \left[\mathbf{x}_{i-1},\mathbf{x}_i\right]^T = \mathbf{A}_{i-1,i}\otimes\left[\mathbf{x}_{i-1},\mathbf{x}_i\right]^T \oplus \mathbf{B}_{i-1,i}\otimes\mathbf{u}_{i-1} \\ \mathbf{y}_i = \mathbf{C}_{i-1,i}\otimes\left[\mathbf{x}_{i-1},\mathbf{x}_i\right]^T \oplus \mathbf{D}_{i-1,i}\otimes\mathbf{u}_{i-1} \end{cases} \quad (13)$$

Using the representation of vector-matrix, the state equation and output equation model can also be rewritten as:
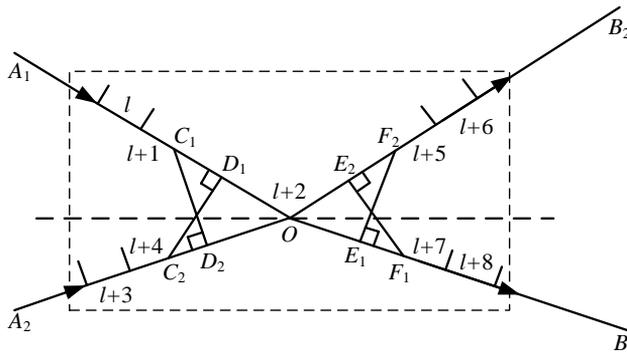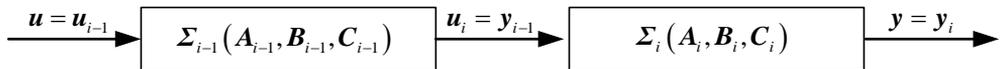

Fig. 2. The division of cross jet routes

Fig.3 The series connection of sub-models

$$\begin{bmatrix} \mathbf{x}_{i-1} \\ \mathbf{x}_i \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{i-1} & \mathbf{0} \\ \mathbf{B}_i \otimes \mathbf{C}_{i-1} \otimes \mathbf{A}_{i-1}, \mathbf{A}_i \end{bmatrix} \otimes \begin{bmatrix} \mathbf{x}_{i-1} \\ \mathbf{x}_i \end{bmatrix} \oplus \begin{bmatrix} \mathbf{B}_{i-1} \\ \mathbf{B}_i \otimes \mathbf{C}_{i-1} \otimes \mathbf{B}_{i-1} \end{bmatrix} \otimes \mathbf{u}_{i-1} \tag{14}$$

$$\mathbf{y}_i = \begin{bmatrix} \mathbf{C}_i \otimes \mathbf{B}_i \otimes \mathbf{C}_{i-1} \otimes \mathbf{A}_{i-1} \\ \mathbf{C}_i \otimes \mathbf{A}_i \end{bmatrix}^T \otimes \begin{bmatrix} \mathbf{x}_{i-1} \\ \mathbf{x}_i \end{bmatrix} \oplus \left(\mathbf{C}_i \otimes \mathbf{B}_i \otimes \mathbf{C}_{i-1} \otimes \mathbf{B}_{i-1} \right) \otimes \mathbf{u}_{i-1} \tag{15}$$

Similarly, if the series connection includes n max-plus sub-models indexed from $(i-1)$ until $(i+n-2)$, then the system matrix, input matrix, output matrix and direct transmission matrix can also be acquired

5. Formulation of multi-aircraft trajectory optimization model

Assuming that the air traffic controllers don't change the expected departure time of all the aircrafts which are involved in the conflict, then their flight speed in the jet route should be changed. Usually, the air traffic controllers prefer ground holding to air holding in practice in order to reduce the workload. As a result, they tend to adopt the method of adjusting the departure time of aircraft or by controlling the arriving time at a specified boundary point. It will inevitably affect other aircrafts in the same jet route or the adjacent jet route, no matter what kind. Besides, the air traffic controllers generally hand over aircrafts according to the predetermined fixed time interval in the specific location (such as termination point of the jet route or exit point of the sector) in order to enhance the ordering of air traffic flow. It is easy to conclude that the adjustment of aircraft arriving time at the jet route entrance corresponds to the parameter $\mathbf{u}$ inherently and the variation of aircraft flight speed corresponds to the parameters $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$. In addition, we can define a cost criterion J that reflects the reference tracking error J_{out} and the control effort J_{in} in the event period $\left[k, k+N_p - 1 \right]$ based on model predictive control theory at present time instant k :

$$J(k) = J_{out}(k) + \lambda J_{in}(k) = \\ = \sum_{j=0}^{N_p-1} \sum_{i=1}^{n_y} \kappa_i(k+j) + \lambda \sum_{j=0}^{N_p-1} \sum_{l=1}^{n_u} u_l(k+j) \tag{16}$$

where N_p is the prediction horizon and λ is a weighting parameter, n_y and n_u denote the number of output variables and control variables.

If we want to reduce the output delay of aircrafts and make the aircrafts arrive the designated location as early as possible, this leads to:

$$J_{out,1} = \sum_{j=0}^{N_p-1} \sum_{i=1}^{n_y} \hat{y}_i(k+j|k) \tag{17}$$

where $\hat{y}_i(k+j|k)$ is the estimate of the output signal $y_i(k+j)$ at time step $(k+j)$ based on the knowledge available at time step k.

If the due dates $r_i(k+j)$ for the finished aircrafts are known and we have to pay a penalty for every delay, a well-suited cost criterion is the tardiness:

$$J_{out,2} = \sum_{j=0}^{N_p-1} \sum_{i=1}^{n_y} \left(\hat{y}_i(k+j|k) - r_i(k+j), 0 \right) \tag{18}$$

Besides, a straightforward translation of the input cost criterion would lead to a minimization of the input time instants and this corresponds to:

$$J_{in,1} = \sum_{j=0}^{N_p-1} \sum_{l=1}^{n_u} u_l(k+j) \tag{19}$$

If we want to balance the input rates, we could take:

$$J_{in,2} = \sum_{j=1}^{N_p-1} \sum_{l=1}^{n_u} \left| \Delta^2 u_l(k+j) \right| \tag{20}$$

where $\Delta u(k+j) = u(k+j) - u(k+j-1)$. Just as in model predictive control for conventional discrete time linear system, we can consider the linear constraint such as:

$$\begin{cases} a_1(k+j) \le \Delta u(k+j-1) \le b_1(k+j) \\ a_2(k+j) \le u(k+j) \le b_2(k+j) \\ a_3(k+j) \le \Delta \hat{y}(k+j|k) \le b_3(k+j) \\ a_4(k+j) \le \hat{y}(k+j|k) \le b_4(k+j) \end{cases} \quad (21)$$

If we combine the material of previous subsections, we finally obtain the following optimization problem (de Schutter & Van den Boom, 2002):

$$\min_{\tilde{u}(k)} \left\{ \alpha J_{in,i}(k) + \beta J_{out,j}(k) \right\} \quad (22)$$

Here α and β represent the weight coefficients of control effort and tracking error, $\alpha + \beta = 1 (\alpha \ge 0, \beta \ge 0)$. By selecting appropriate value of α and β, various de-confliction trajectory optimization models can be obtained.

6.   Worked example

Without loss of generality, three cross flight aircraft trajectory planning are demonstrated using the cost function $J_{out,1}$ in order to verify the validity of the model presented above. The initial speeds of the three aircrafts are $v_1 = 620km/h$, $v_2 = 800km/h$ and $v_3 = 920km/h$. In addition, the aircraft flow is controlled by the output variable and the flight speed range of each aircraft is $[600km/h, 960km/h]$. The sample jet route spatial configuration is shown in Fig.4. Aircraft 1 and aircraft 2(simply "AC2" for short in the following section) sink to segment AB from the upper left and lower left respectively while aircraft 3(simply"AC3" for short in the following section) sinks to segment AB from left. In addition, O_1 and O_2 denote two jet routes intersections.
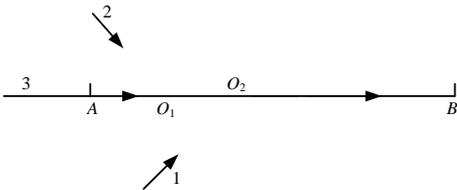

Fig.4. The spatial configuration of cross jet routes

Besides, the locations of aircraft 2 and aircraft 3 are O_1 and A respectively when aircraft 1 arrives at O_2. Moreover, $d_{min} = 10km$, $|AO_1| = |O_1O_2| = 30km$, $|O_2B| = 90km$ and the terminate point of trajectory planning is B. For jet route AB, the service precedence of three aircrafts is aircraft 1, aircraft 2 and aircraft 3. Due to the speed difference of aircraft, if we view the time instant when aircraft 1 arrives at O_2 as the original 0 time instant, then the horizontal separation between aircraft 1, aircraft 2 and aircraft 3 in future time interval shall not meet the minimum safety distance defined above. Therefore, it is necessary to adjust the flight parameter of relevant aircrafts.

In the following section, we will mainly discuss how to adjust the aircraft arriving time at the jet route entry to get de-confliction aircraft trajectory. Furthermore, the aircraft trajectory planning algorithms can be categorized into deterministic case, random case and model predictive control case considering the random factors. Each case includes the trajectory planning result of "Non-relaxed" and "Relaxed". The difference between the two results lies in the buffer time t_k. For the random case and model predictive control case, the value of $t_i(k)$ conform to the law of normal distribution $N(1/v*360, 5)$ and $v(km/h)$ is the corresponding aircraft velocity. Besides, the time step of the simulation is 4s and $N_p = 8$, $N_c = 5$. The minimum time that each aircraft should delay and the optimal output criterion are given in Tab.1 for the three different cases.

Table 1. The simulation result for the three different cases

Random case				Model predictive control case	
Non-relaxed		Relaxed		Non-relaxed	Relaxed
AC2	AC3	AC2	AC3	$J_{out,1}$	$J_{out,1}$
129s	222s	117s	213s	1820s	1759s

Furthermore, it is worth emphasizing that the adjustment of arriving time at the jet route entrance and flight speed on the jet route can also be used in combination. In addition, from the proposed system model, the readers can also see that only adjusting the aircraft flight speed on the jet route may not meet the minimum safety separation because of the physical performance of the aircraft and the initial aircraft relative position on the jet route. However, it is always feasible by adjusting the arriving time at the jet route entrance, despite the delay time is very long sometimes.

7. Conclusion

This paper presented a max-plus model that can be used to develop robust optimal control policies for the air traffic flow and the reason for using model predictive control approach for max-plus system is the same as for conventional linear systems. The system model takes into account all air traffic resources, including jet routes and aircrafts. The multiple aircraft conflict-free strategic trajectory optimization model is very flexible for structure changes (since the optimal strategy is recomputed every time step or event step so that model changes can be taken into account as soon as they are identified) and it can provide decision support for air traffic controllers to control traffic flow rates at the control boundaries of their choice. We present the simulation results for the performance of control strategies with different information requirements and believe that the proposed model is particularly useful in developing planning strategies at the strategic level, where coordination of airspace resources at distinct locations is necessary. Topics for future research include the influences of the tuning parameters in the presented model.

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